Quiz 3

60 Minutes

First Name: SOLUTIONS

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

For [**Show Work**] problems, you **MUST** show correct work to get credit.
Correct answers with no explanation will get a 0.

Final Score: _____ / 200

1. Consider the sets $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Which function is an injection from A to B?

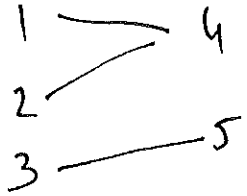
   [A] $f(1) = 4, f(2) = 4, f(3) = 5$
   [B] $f(1) = 4, f(2) = 5, f(3) = 5$
   [C] $f(1) = 4, f(2) = 4$
   [D] $f(1) = 4, f(3) = 5$
   [E] None of the above.

$|A| > |B|$
There is no injection from A to B.

2. Consider the sets $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Which function is a surjection from A to B?

   [A] $f(1) = 4, f(2) = 4, f(3) = 5$
   [B] $f(1) = 5, f(2) = 5, f(3) = 5$
   [C] $f(1) = 4, f(2) = 4$
   [D] $f(1) = 5, f(3) = 5$
   [E] None of the above.

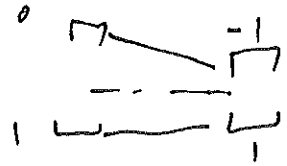
Not functions



3. Consider the real intervals $A = [0, 1]$, $B = [-1, 1]$. Which function is a bijection from A to B?

   [A] $f(x) = 2x + 2$
   [B] $f(x) = 2x + 1$
   [C] $f(x) = 2x - 1$
   [D] $f(x) = 2x$
   [E] None of the above.

Take any $a \in A$.
$f(a) = 2a - 1 \in [-1, 1]$



4. Consider the real interval $A = [0, 1]$ and the set B of all *rationals* in the interval $[-1, 1]$. Which function is a bijection from A to B?

   [A] $f(x) = 2x + 2$
   [B] $f(x) = 2x + 1$
   [C] $f(x) = 2x - 1$
   [D] $f(x) = 2x$
   [E] None of the above.

Reals are uncountable.
Rationals are countable.
There is no bijection.

5. Which of the following sets is countable (recall $\mathbb{N}$ is natural numbers, $\mathbb{Q}$ is rationals)?

   [A] The set of all functions from $\mathbb{N}$ to $\mathbb{N}$ ← infinite binary strings
   [B] The set of all functions from $\mathbb{N}$ to $\mathbb{Q}$ ← infinite binary strings
   [C] The set of all infinite binary strings
   [D] The set of all pairs of natural numbers (m, n), where $m, n \in \mathbb{N}$
   [E] None of the above

Listing: $\{(1,1), (1,2), (2,1), (2,2) \ldots\}$

6. How do we know the set of all functions is uncountable?

- [A] The set of all programs is uncountable.
- [B] There is a bijection from the set of all binary functions defined on $\mathbb{N}$ to the set of all infinite binary strings.
- [C] The set of finite binary strings is uncountable.
- [D] The set of all functions is countable.
- [E] None of the above.

7. [**Show Work**] How many surjections from $\{1,2,3,4,5\}$ to $\{1,2\}$ are there?

- [A] 10
- [B] 20
- [C] 30
- [D] 32
- [E] None of the above.

Total # of functions: $2^5 = 32$
Of those 2 are not surjections
$f(1) = \cdots = f(5) = 1$
$f(1) = \cdots = f(5) = 2$.

8. What is the relationship between regular languages and DFAs?

- [A] There exist regular languages which cannot be decided by a DFA.
- [B] There exist languages which can be decided by a DFA but cannot be described by a regular expression.
- [C] Regular languages is the set of languages that can be decided by DFAs.
- [D] All languages are regular.
- [E] None of the above.

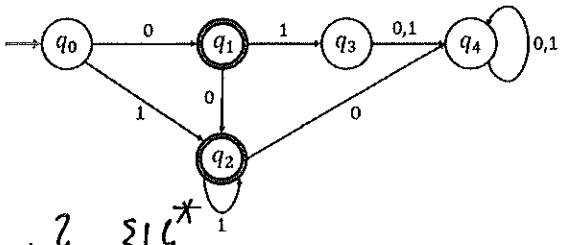
9. Consider the language $\mathcal{L} = \{1, 01\}^*$. Which string is not in $\mathcal{L}$?

- [A] 11 $1 \cdot 1$
- [B] 011 $01 \cdot 1$
- [C] 010101 $01 \cdot 01 \cdot 01$
- [D] 10101011 $1 \cdot 01 \cdot 01 \cdot 01 \cdot 1$
- [E] They are all in $\mathcal{L}$.

10. Consider the language $\mathcal{L} = \{0^{\bullet n} 1^{\bullet 3n} \mid n \geq 0\}^*$. What do we know about $\mathcal{L}$?

- [A] $\mathcal{L}$ contains all strings with three times as many 1s as 0s.
- [B] $\mathcal{L}$ contains all strings that start with a 0.
- [C] $\mathcal{L}$ is regular. similar to $0^n 1^n$
- [D] $\mathcal{L}$ is finite.
- [E] None of the above.

11. Consider the deterministic finite automaton (DFA) below. What is the language decided by this machine?



A $\mathcal{L} = \{0, 00, 1\}$

B $\mathcal{L} = \{0, 00, 1\}^*$

C $\mathcal{L} = \{0, 00\} \cup \{1\}^*$

D $\mathcal{L} = \{\varepsilon, 0, 00\} \cup \{1\}^*$

E None of the above.

$\{00, 1\} \cdot \{1\}^*$

12. Consider the DFA in Question 11. Which state does the DFA terminate in for the input string 0010?

$q_0 \xrightarrow{0} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_2 \xrightarrow{0} q_4$

A q_1

B q_2

C q_3

D q_4

E None of the above

13. Consider the language $\mathcal{L} = \{01\}^*\{0\}^*\{10\}^*$. Which word is not in $\mathcal{L}$?

A 0110 $01 \cdot \varepsilon \cdot 10$

B 011 $01 \cdot ?$

C 010 $01 \cdot 0 \cdot \varepsilon$

D ε $\varepsilon \cdot \varepsilon \cdot \varepsilon$

E They are all in $\mathcal{L}$.

14. [**Show Work**] Which of the following languages cannot be solved by a DFA?

A $\mathcal{L}$ = {strings with at least three 1s and at least three 0s} $* \cdot \{0,1\}^3 \cdot *$

B $\mathcal{L}$ = {strings with at least five million 0s} $* 0 * 0 \cdots * 0 *$ (5M)

C $\mathcal{L}$ = {strings with no 1s and no 0s} ε

D $\mathcal{L}$ = {strings of even length} $\{00, 01, 10, 11\}^*$

E They are all regular languages.

15. Which of the following strings *cannot* be generated by the CFG: $S \to \varepsilon \mid 1S \mid 0S$?

A 1001

B 0011

C 1100

D 1010

E They can all be generated.

all strings

Induction on derivation length

16. [**Show Work**] Which CFG generates the language of all strings of odd length?

- [A] $S \to \varepsilon \mid 1S0 \mid 0S1$
- [B] $S \to \varepsilon \mid 1S \mid 0S$
- [C] $S \to 0 \mid 1 \mid 1S0 \mid 0S1 \mid 0S0 \mid 1S1$
- [D] $S \to 0 \mid 1 \mid 1S0S \mid S0S1$
- [E] None of the above.

All strings generated by CFG are odd-l:
Base case: 0 or 1. ✓
Inductive step: Each rule looks like ISD has shorter derivation.

Induction on string length
Base case: 0 or 1. ✓ Inductive step: each string $w = 1w_0 0$ or $w = 0w_0 1$ or...

All odd-length strings generated by CFG:
Strong Induction on w_0. hypothesis

17. Which of the following regular expressions *cannot* be generated by a CFG?

- [A] $1 * 0$
- [B] $(01 * 1 * 0)^*$
- [C] $\overline{\{0,1\}}$
- [D] $\{0,1\}^*$
- [E] All regular expressions can be generated by a CFG.

18. What is the relationship pushdown automata (PDAs) and Turing Machines (TMs)?

- [A] They can decide the same set of languages.
- [B] TMs can decide some languages that no PDA can decide.
- [C] PDAs can decide some languages that no TM can decide.
- [D] They are the same model.
- [E] None of the above.

19. Each TM description can be written as a finite binary string. What do we know?

- [A] The number of TMs is uncountable.
- [B] The number of TMs is the same as the number of infinite binary strings.
- [C] The number of TMs is countable.
- [D] There is a bijection between the set of TMs and the set of infinite binary strings.
- [E] None of the above.

20. Which statement is true about the language $\mathcal{L} = \{w\#w^R \mid w \in \{0,1\}^*\}$?

- [A] A DFA can solve this language. A TM can solve this language.
- [B] A DFA cannot solve this language. A PDA can solve this language.
- [C] A PDA can solve this language. A TM cannot solve this language.
- [D] A PDA cannot solve this language. A TM cannot solve this language.
- [E] None of the above.

Scratch