Deterministic Policy Gradients

Reading

- Silver, David, Guy Lever, Nicolas Heess, Thomas Degris, Daan Wierstra, and Martin Riedmiller. "Deterministic policy gradient algorithms." In *International conference on machine learning*, pp. 387-395. PMLR, 2014.
- Lillicrap, Timothy P., Jonathan J. Hunt, Alexander Pritzel, Nicolas Heess, Tom Erez, Yuval Tassa, David Silver, and Daan Wierstra. "Continuous control with deep reinforcement learning." arXiv preprint arXiv:1509.02971 (2015).
- Fujimoto, Scott, Herke Hoof, and David Meger. "Addressing function approximation error in actor-critic methods." In International conference on machine learning, pp. 1587-1596. PMLR, 2018.

Overview

- Q-learning works well for systems with finitely many actions
- However, most real-world systems have infinitely many control actions
  - One could discretize the action space and still use Q-learning
  - Unlikely to scale if we have more than a couple of dimensions to discretize over
- Need to extend Q-learning methods to infinite-state MDPs

Extending Q-learning to infinite action spaces

- First, suppose we know the optimal Q function
  - in an infinite action space
- Can compute the policy π by maximizing Q over all actions:
  $$\pi(s) = \max_a Q(s,a)$$
  - May be hard if Q is a complex function (e.g., a neural net)
  - But would probably find a good local optimum eventually
- Of course, we don't know the optimal Q
  - Need to iteratively update Q and π
- Could apply the Q-learning iteration followed by the maximization above
  - Research in the past showed that this approach is quite unstable

Deterministic Policy Gradient

- Standard policy gradient theorem is very inefficient (why?)

$$\nabla v_\pi(s_0) = \sum_s d_\pi(s) \sum_a \nabla \pi(a|s) q_\pi(s,a)$$

  – Need to know/approximate q_π for all actions
  – Hard to do in an infinite space
- One can avoid the expectation over all actions by introducing a deterministic policy gradient
- For a deterministic policy, there is only one action per state
  – Most policies are deterministic anyway, e.g., neural nets

[1]David Silver, Guy Lever, Nicolas Heess, Thomas Degris, Daan Wierstra, and Martin Riedmiller. "Deterministic policy gradient algorithms." In *International conference on machine learning*, pp. 387-395. PMLR, 2014.

Deterministic Policy Gradient, cont'd

- Standard policy gradient theorem is very inefficient

$$\nabla v_\pi(s_0) = \sum_s d_\pi(s) \sum_a \nabla \pi(a|s) q_\pi(s, a)$$

- However, stochastic policies have a crucial benefit
  - Exploration!
- If target policy is deterministic, need a stochastic behavior policy
  - And an off-policy algorithm!

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Deterministic Policy Gradient Theorem

- What would be different from the stochastic version:

$$\nabla_{\boldsymbol{\theta}} v_\pi(s_0) = \sum_s d_\pi(s) \sum_a \nabla_{\boldsymbol{\theta}} \pi(a|s) q_\pi(s,a)$$

  - No need to average over all actions any more
  - Can evaluate gradient at the specific action taken by the policy

- Deterministic policy gradient theorem:

$$\nabla_{\boldsymbol{\theta}} v_\pi(s_0) = \int_S d_\pi(s) \nabla_{\boldsymbol{\theta}} \pi(s;\boldsymbol{\theta}) \nabla_a q_\pi(s,a)\Big|_{a=\pi(s)} ds$$

  - Policy gradients for deterministic policies only make sense over continuous spaces (why?)
  - The q-value function is not differentiable w.r.t. a otherwise

Continuous-space Markov Decision Processes

- Before we look at the deterministic policy gradient proof, we need to talk about continuous-space MDPs
- How do we define an infinite-state MDP?
  - It is a 5-tuple (S, A, P, R, η) as before
  - Now S and A are infinite
  - Infinite spaces can be tricky but a standard choice is to use a probability density function (pdf)
  - The transition function is described with a pdf $p(s'|s, a)$

    $$\mathbb{E}[S_{t+1}|S_t = s, A_t = a] = \int s' p(s'|s, a) ds'$$

    - Recall that for any pdf $\int p(s'|s, a) ds' = 1$
  - The reward function R is defined similarly

    $$R_e(s, a) = \mathbb{E}[R_{t+1}|S_t = s, A_t = a] = \int R(s', s, a) p(s'|s, a) ds'$$
  - The initial condition η is also a pdf

Continuous-state Bellman Equation

- Turns out the Bellman equation is the same for continuous-state MDPs as it is for finite-state ones

$$\begin{aligned} q_\pi(s,a) &= \mathbb{E}_\pi[R_{t+1} + \gamma v(S_{t+1}) | S_t = s, A_t = a] \\ &= \int \big(R(s', a, s) + \gamma v(s')\big) p(s' | s, a) ds' \end{aligned}$$

  - Don’t have time to prove

- If π is deterministic, what is a?
  - It’s just $\pi(s)$

$$v_\pi(s) = q_\pi\big(s, \pi(s)\big) = \int \big(R(s', \pi(s), s) + \gamma v(s')\big) p(s' | s, \pi(s)) ds'$$

Deterministic Policy Gradient Theorem Proof

- Fairly similar to the stochastic case, with some differences

$$\nabla_{\boldsymbol{\theta}} v_\pi(s) = \nabla_{\boldsymbol{\theta}}[q_\pi(s, \pi(s))]$$

$$= \nabla_{\boldsymbol{\theta}} \int_S p(s'|s, \pi(s))[R(s', \pi(s), s) + \gamma v_\pi(s')]\, ds'$$

chain rule

$$= \int_S \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a [p(s'|s, a)\, R(s', a, s)] \Big|_{a=\pi(s)} ds' + \nabla_{\boldsymbol{\theta}} \int_S p(s'|s, \pi(s)) \gamma v_\pi(s')\, ds'$$

$$= \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a R_e(s, a) \Big|_{a=\pi(s)} + \nabla_{\boldsymbol{\theta}} \int_S p(s'|s, \pi(s)) \gamma v_\pi(s')\, ds'$$

$$= \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a R_e(s, a) \Big|_{a=\pi(s)} +$$

Leibniz rule + chain rule

$$\gamma \int_S p(s'|s, \pi(s)) \nabla_{\boldsymbol{\theta}} v_\pi(s') + \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a p(s'|s, a) \Big|_{a=\pi(s)} v_\pi(s') ds'$$

$$= \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a \left(R_e(s, a) + \gamma \int_S p(s'|s, a) v_\pi(s') ds' \right) \Big|_{a=\pi(s)} +$$

$$\gamma \int_S p(s'|s, \pi(s)) \nabla_{\boldsymbol{\theta}} v_\pi(s')\, ds'$$

Deterministic Policy Gradient Theorem Proof, cont'd

$$\nabla_{\boldsymbol{\theta}} v_\pi(s) =$$

$$= \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a \left(R_e(s,a) + \gamma \int_S p(s'|s,a) v_\pi(s') ds' \right) \Big|_{a=\pi(s)} +$$

$$\gamma \int_S p(s'|s,\pi(s)) \nabla_{\boldsymbol{\theta}} v_\pi(s')\, ds'$$

$$= \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} + \gamma \int_S \mathbb{P}_\pi[s \to s', 1] \nabla_{\boldsymbol{\theta}} v_\pi(s')\, ds'$$

- Does this look familiar?
  - Same sequence as stochastic policy gradient theorem

Deterministic Policy Gradient Theorem Proof, cont'd

$$
\begin{aligned}
\nabla_{\boldsymbol{\theta}} v_\pi(s) = {} & \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} + \\
& + \gamma \int_S \mathbb{P}_\pi[s \to s', 1] \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} ds' \\
& + \gamma^2 \int_S \mathbb{P}_\pi[s \to s', 2] \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} ds' \\
& + \cdots \\
= {} & \sum_{t=0}^{\infty} \int_S \gamma^t \mathbb{P}_\pi[s \to s', t] \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} ds' \\
= {} & \int_S \sum_{t=0}^{\infty} \gamma^t \mathbb{P}_\pi[s \to s', t] \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} ds' \quad \text{Fubini's theorem}
\end{aligned}
$$

Deterministic Policy Gradient Theorem Proof, cont'd

$$\nabla_{\boldsymbol{\theta}} v_\pi(s) = \int_S \sum_{t=0}^{\infty} \gamma^t \mathbb{P}_\pi[s \to s', t] \nabla_{\boldsymbol{\theta}} \pi(s) \nabla_a q(s,a) \Big|_{a=\pi(s)} ds'$$

- Similar to the stochastic case, we have a discounted aggregate visitation function $d_\pi(s)$
  - Now it is a pdf, not a probability function, since we have a continuous space

$$d_\pi(s') = \sum_{t=0}^{\infty} \gamma^t \mathbb{P}_\pi[s \to s', t]$$

  - where

$$\mathbb{P}_\pi[s \to s', t] = \int p(s_1|s, \pi(s)) \int p(s_2|s_1, \pi(s_1)) \ldots \int p(s'|s_{t-1}, \pi(s_{t-1})) ds_{t-1} \ldots ds_2 ds_1$$

- Plugging d_π back in gives the final result

$$\nabla_{\boldsymbol{\theta}} v_\pi(s_0) = \int_S d_\pi(s) \nabla_{\boldsymbol{\theta}} \pi(s; \boldsymbol{\theta}) \nabla_a q_\pi(s,a) \Big|_{a=\pi(s)} ds$$

Off-Policy Deterministic Actor-Critic

- Computing the deterministic gradient is easier
  - So far, so good
- But learning using a deterministic policy is harder
  - Not much exploration with a deterministic policy
- Need a behavior policy b
  - And an off-policy learning approach
  - What is the challenge with such an approach?
  - Data is generated from b, need to adapt the policy gradient theorem

Stochastic Off-Policy Actor-Critic

- Since the data is generated from b we need to consider the quantity

$$J(\boldsymbol{\theta}) = \mathbb{E}_b\left[v_{\pi_{\boldsymbol{\theta}}}(S_t)\right]$$

- In the finite case the expectation is expanded as

$$\begin{aligned} J(\boldsymbol{\theta}) &= \sum_s d_b(s) v_{\pi_{\boldsymbol{\theta}}}(s) \\ &= \sum_s d_b(s) \sum_a \pi(a|s) q_{\pi_{\boldsymbol{\theta}}}(s,a) \end{aligned}$$

  – where d_b is the discounted aggregate visitation probability

- We want to pick $\boldsymbol{\theta}$ that maximizes $J(\boldsymbol{\theta})$
  – As usual, we'll use gradient ascent

Degris, Thomas, Martha White, and Richard S. Sutton. "Off-policy actor-critic." arXiv preprint arXiv:1205.4839 (2012).

Stochastic Off-Policy Actor-Critic, cont'd

- In the finite case the expectation is expanded as

$$J(\boldsymbol{\theta}) = \sum_s d_b(s) \sum_a \pi(a|s) q_{\pi_{\boldsymbol{\theta}}}(s,a)$$

- Look at the gradient (using chain rule)

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) = \sum_s d_b(s) \sum_a \nabla_{\boldsymbol{\theta}} \pi(a|s) q_{\pi_{\boldsymbol{\theta}}}(s,a) + \sum_s d_b(s) \sum_a \pi(a|s) \nabla_{\boldsymbol{\theta}} q_{\pi_{\boldsymbol{\theta}}}(s,a)$$

- Paper argues that minimizing first term is enough
  - Shows proof in case of finite-state case
    - Infinite-case proof/claim has some issues

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \approx \sum_s d_b(s) \sum_a \nabla_{\boldsymbol{\theta}} \pi(a|s) q_{\pi_{\boldsymbol{\theta}}}(s,a)$$

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Off-Policy Deterministic Actor-Critic, cont'd

- Deterministic paper uses the same off-policy gradient approximation as in Sutton paper

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \approx \int_S d_b(s) \nabla_{\boldsymbol{\theta}} \pi(s; \boldsymbol{\theta}) \nabla_a q_\pi(s, a) \Big|_{a=\pi(s)} ds$$

  - Approximation quality is not clear but ultimately with modern RL algorithms the proof is in the pudding
- Crucially, this approximation means that one can apply the on-policy algorithm in the off-policy setting
  - This seems suspicious, but if the approximation is good enough locally, then algorithm may work well
    - Given small enough learning rate and other stabilization techniques used in modern RL

Off-Policy Critic Training

- Since trajectories are generated by behavior policy b, we can't train the critic on-policy

$$\delta_t = R_t + \gamma Q^{\boldsymbol{w}}(S_{t+1}, A_{t+1}) - Q^{\boldsymbol{w}}(S_t, A_t)$$

  - The A_{t+1} would have been generated by b
- How did we modify TD-learning to make it off-policy?
  - Q-learning!
  - Can't compute max anymore. What is the alternative?

$$\delta_t = R_t + \gamma Q^{\boldsymbol{w}}\big(S_{t+1}, \pi(S_{t+1})\big) - Q^{\boldsymbol{w}}(S_t, A_t)$$

- The rest of the updates are the same as in the on-policy case

$$\boldsymbol{w}' = \boldsymbol{w} + \alpha_{\boldsymbol{w}} \delta_t \nabla_{\boldsymbol{w}} Q^{\boldsymbol{w}}(S_t, A_t)$$

$$\boldsymbol{\theta}' = \boldsymbol{\theta} + \alpha_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}} \pi_{\boldsymbol{\theta}}(s) \nabla_a Q^{\boldsymbol{w}}(s, a) \Big|_{a=\pi(s)}$$

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Overview

- Combine classical actor-critic ideas with new insights from the DQN paper
- Use neural networks for both the actor and critic and update them using (deterministic) policy gradients
- Stabilize learning with recent insights such as experience replay and target networks (explained later)
- Also add batch normalization
- Method called deep deterministic policy gradient (DDPG)
  - One of the early breakthroughs in continuous-action deep RL

Setup

- Environment is an MDP, as usual
  - No finite-state assumption necessary
  - Different environments considered, ranging from standard control tasks to RL benchmarks such as the pendulum
- Action space is continuous (possibly multidimensional)
- Observe a reward R_t at each time t
- The goal is to learn a (deterministic) policy π that maps the current state to a control action, maximizing the expected (discounted) reward

Training the Critic

- Critic is a neural network, trained using supervised learning
- Recall the iteration
  - After observing a tuple $(S_t, A_t, R_t, S_{t+1}, A_{t+1})$, compute the difference $\delta_t = R_t + \gamma Q^{\boldsymbol{w}}\big(S_{t+1}, \pi(S_{t+1})\big) - Q^{\boldsymbol{w}}(S_t, A_t)$
  - Let $y = R_t + \gamma Q^{\boldsymbol{w}}\big(S_{t+1}, \pi(S_{t+1})\big)$. Then minimize loss:
    $$L_{\boldsymbol{w}}(Q^{\boldsymbol{w}}, S_t, A_t, y) = (y - Q^{\boldsymbol{w}}(S_t, A_t))^2$$
  - i.e., follow gradient $2\delta_t \nabla_{\boldsymbol{w}} Q^{\boldsymbol{w}}(S_t, A_t)$:
    $$\boldsymbol{w}' = \boldsymbol{w} + \alpha_{\boldsymbol{w}} \delta_t \nabla_{\boldsymbol{w}} Q^{\boldsymbol{w}}(S_t, A_t)$$
  - Can train directly using gradient ascent

Training the Actor

- The actor is a neural network that takes in the observation/state and outputs the control action
- Unlike DQN, this network is not a classifier, but a regressor
- Output layer has one neuron (for each action dimension)
- Trained using the deterministic policy gradient

$$\boldsymbol{\theta}' = \boldsymbol{\theta} + \alpha_{\boldsymbol{\theta}} \nabla_{\boldsymbol{\theta}} \pi(S_t) \nabla_a Q^{\boldsymbol{w}}(S_t, a) \Big|_{a=\pi(S_t)}$$

Target Networks

- Recall the idea in double Q-learning
  - Use two Q estimators, one to pick the maximal action and one to estimate the bootstrapped return
  - Reduces maximization bias
- Normally, $Q^{\boldsymbol{w}}$ and $\pi_{\boldsymbol{\theta}}$ networks used to bootstrap the labels y

$$y = R_t + \gamma Q^{\boldsymbol{w}}\big(S_{t+1}, \pi_{\boldsymbol{\theta}}(S_{t+1})\big)$$

  - This adds a lot of variance and slows down training ultimately
- Instead, we could have separate "target" networks $Q^{\boldsymbol{w}'}$ and $\pi_{\boldsymbol{\theta}'}$ to bootstrap the labels
  - We update these more slowly than the ones we train

Target Networks, cont'd

- Target $\boldsymbol{\theta}_t$ updated as a weighted average of previous target parameters and trained parameters $\boldsymbol{\theta}$

$$\boldsymbol{\theta}_t' = \tau\boldsymbol{\theta} + (1 - \tau)\boldsymbol{\theta}_t$$

  – where τ is close to 0
  – Same for $\boldsymbol{w}$ and $\boldsymbol{w}'$
- This process greatly stabilizes training
  – It may make it slower in some cases but the benefits in stability outweigh the cost in noise without targets
  – It is a low-pass filter of sorts

Exploration

- A common and simple way to enforce exploration is to add noise to the actions
- The choice of noise is essential because sometimes we need to add similar noise over several steps (e.g., to force a turn)
  - If we add random noise at every step, may not explore enough
- One popular choice is the Ornstein-Uhlenbeck (OU) process

$$\dot{x}(t) = -\theta x(t) + \sigma \eta(t)$$

  - where θ and σ are parameters and η is Gaussian noise
  - this is effectively a random walk, where θ and σ determine the mean and variance, respectively
  - correlation over time ensures that non-trivial actions can be explored over time

Full DDPG algorithm

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N} \sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu} \mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$

$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

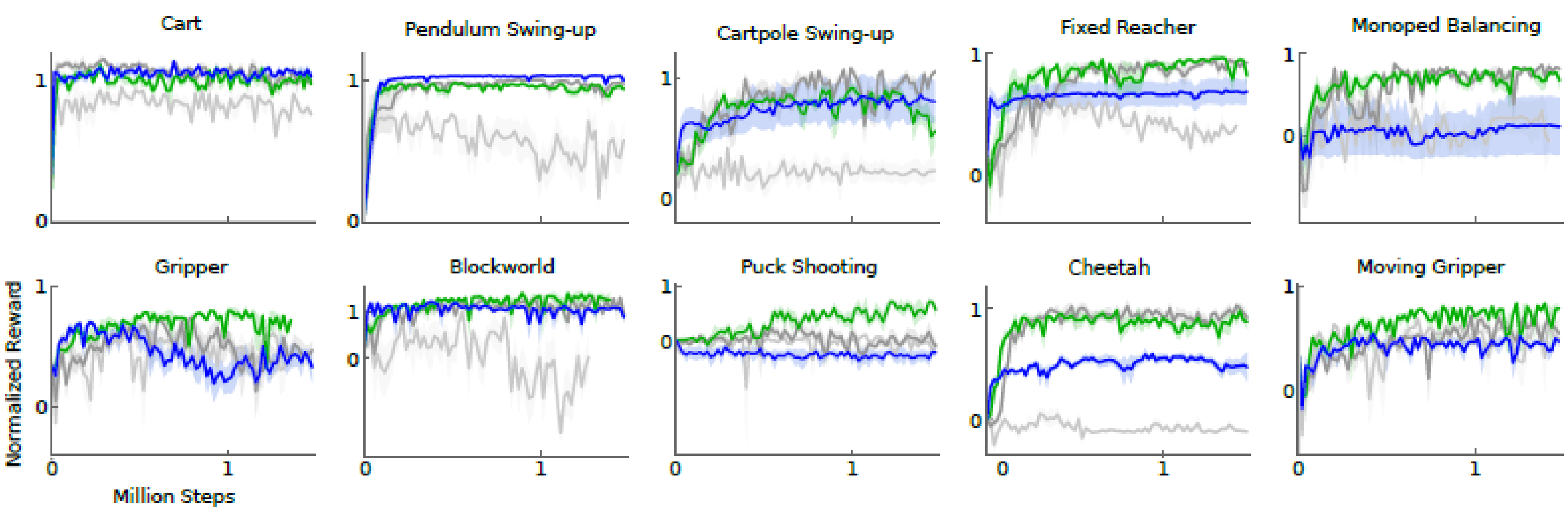
  end for
end for

Experimental environments

- Authors looked at a number of control tasks
- Classical benchmarks such as cartpole, pendulum
- MuJoCo environment tasks
  - cheetah, monoped, locomotion tasks
- Torcs (driving simulator)

Results

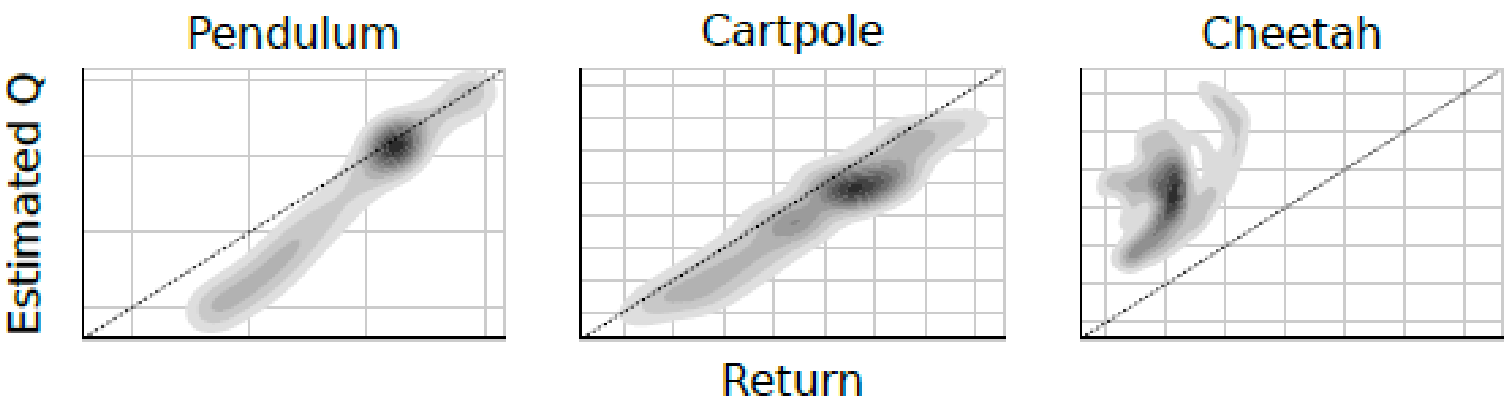
- DDPG much more stable than pure DPG on almost all tasks
  - Target networks and batch norm seem to be a good combo
- Can also learn fairly good policies directly from pixels (blue)



Light grey: original DPG
Dark grey: DPG + batch norm
Green: DPG + batch norm + target networks (DDPG)
Blue: raw pixel data

Accuracy of estimated Q-values

- Estimates reasonably accurate for simpler tasks
- Overestimation issues for complex tasks (e.g., cheetah)
- This is a known problem in all Q-learning-like methods
  - The overapproximation bias has returned!



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Overview

- Value function overestimation is a common problem in Q-learning
  - Already saw maximization bias in standard Q-learning
- It can degrade the quality of learning in several ways
  - Reduce exploration when a suboptimal action has a greatly overestimated value
  - Increase variance if several suboptimal actions have overestimated values
- Overall, learning is significantly slowed down
  - May not even converge in some cases

Correcting Overestimation Bias

- Target networks in the DDPG paper are an analogue of double Q-learning
  - But not quite double Q-learning since the networks change slowly – target and behavior nets are often similar
  - But true double Q-learning can't be applied to the deep RL setting exactly (why?)
    - Critics are trained on the same buffer (not independent)
- Instead train two separate critics and use the critic that provides the lower value

$$y = r + \gamma \min_{i=1,2} Q_{\boldsymbol{w}_i}\big(s, \pi(s)\big)$$

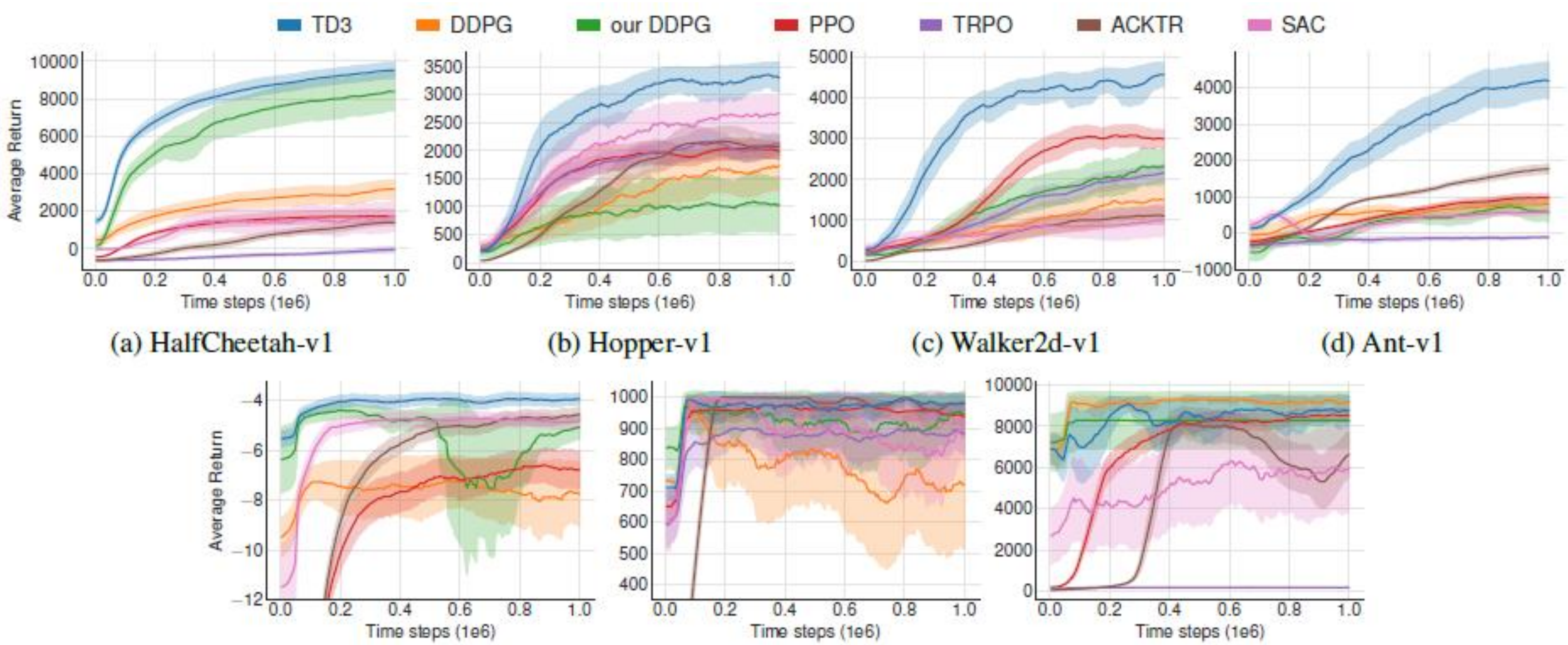
  - Critics are trained in the same way as before (just initialized with different weights)
  - Alleviates the overestimation error

Delayed Policy Updates

- Do not perform the policy gradient every time but rather every T iterations (where T is typically several hundred)
  - Same for target network updates
- What is the benefit of this approach?
  - Policy gradients are really expected values
    - Delaying allows us to collect more data from the current policy
    - Obtain a better estimate of the expectation
  - Also, effectively makes training more on-policy
    - Since we're effectively using the on-policy gradient, this ought to help as well

Results

- Training is much more stable and able to find better policies in most cases
- TD3 is my algorithm of choice for continuous-action RL



(a) HalfCheetah-v1 (b) Hopper-v1 (c) Walker2d-v1 (d) Ant-v1

(e) Reacher-v1 (f) InvertedPendulum-v1 (g) InvertedDoublePendulum-v1