

Foundations of Computer Science

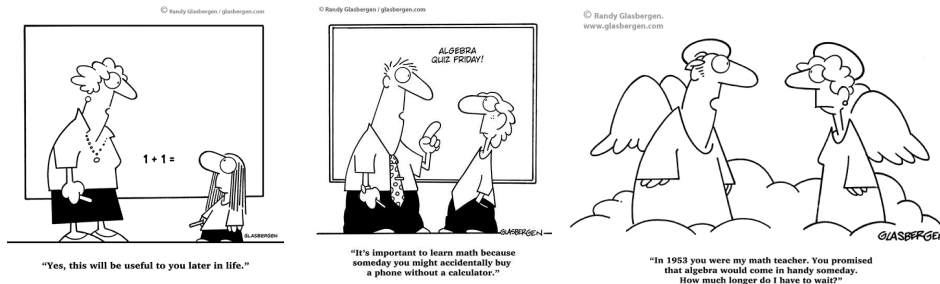
Lecture 1

Warmup: A Taste for Discrete Math and Computing

Background

Disease spread, speed-dating, friendship networks

3 Challenge Problems



(Today) Warmup: A Taste for Discrete Math and Computing

- 1 Resources and Rules
- 2 Storyline
- 3 Background
- 4 A Taste of Discrete Math
 - Two-Contact Ebola on a Grid
 - Scheduling Speed Dates
 - Friendship Networks and Ads
 - Modeling Computers
- 5 Getting Good at Discrete Math
 - Computing is Mathematics
 - Polya's Mouse
- 6 3 Challenge Problems

Resources and Rules

- 1 Web Page: www.cs.rpi.edu/~magdon/courses/focs.php
 - course info: www.cs.rpi.edu/~magdon/courses/focs/info.pdf
 - schedule+reading+slides: www.cs.rpi.edu/~magdon/courses/focs/slides.html
 - assignments+exams: www.cs.rpi.edu/~magdon/courses/focs/assign.html
- 2 Text Book: Discrete Mathematics and Computing (Magdon-Ismail).
- 3 TAs, UG-Mentors.
- 4 Recitation Section.
- 5 ALAC Drop-in-tutoring.
- 6 Professor.
- 7 **Prerequisites:**
 - CS II (data structures)
 - Calc I (Calc II **STRONGLY** recommended)
- 8 **Rules:** No food, no electronics, no cheating.

The Storyline

- 1 Discrete objects.
- 2 Reasoning about discrete objects
- 3 Counting discrete objects
- 4 Randomness: probability
- 5 What can we compute?
- 6 What can we compute efficiently?

- concepts/concrete
- proof/theory/abstract
- theory of computation

our language will be mathematics ...
... it will be everywhere

Background

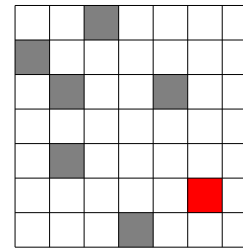
Programming, numbers, geometry, algebra, calculus, ...

- What is the minimum element in the set {8, 9, 3, 10, 19}?
- Does this set of *positive numbers* have a minimum element:
{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, ...}

Any (non-empty) set containing only **positive integers** has a minimum element.

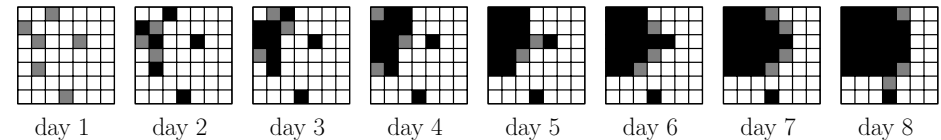
Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.



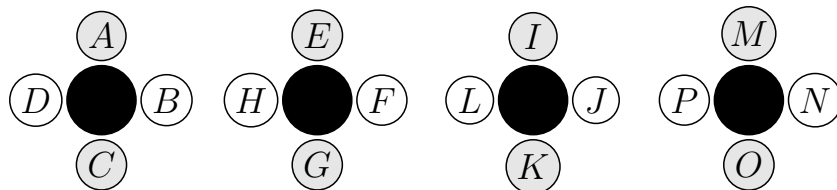
- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?
- Given few vaccines, who to immunize?
- What were the “entry points”?

Answers involve discrete math.



Scheduling Speed Dates

In each round 4 people “group”-speed-date around a table. (4 rounds in all)



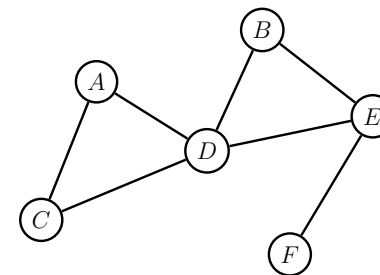
How to organize the rounds so that people meet as many people as possible?

- Do you care about average or minimum number of meetups per person?
- Can everyone meet at least 10 people?
- What happens if you assign tables randomly?

Answers involve discrete math.

Friendship Networks and Ads

People are circles and links are friendships.



Who would you advertise to? You wish to maximize adoption of your new technology.

Answers involve discrete math.

Modeling Computers

Desktop, smartphone, fitbit, ...

What is computing?

We have deep questions:

- 1 What can we compute?
- 2 What *can't* we compute?
- 3 How fast?

Dominos: d_1 d_2 d_3

0	01	110
100	00	11

$d_3 d_1 d_3 =$

110	0	110
11	100	11

 \rightarrow

1100110
1110011

Domino puzzle: Want same top and bottom.

Domino program:

Input: dominos

Output: sequence that works

or

say it can't be done

Answers involve discrete math.

Computing is Mathematics

"Too few people recognize that the high technology so celebrated today is essentially a mathematical technology."

"A programmer must *demonstrate* that their program has the required properties. If this comes as an afterthought, it is all but certain that they won't be able to meet this obligation. Only if this obligation influences the design is there hope to meet it. . .

"The required techniques of effective reasoning are pretty formal, but as long as programming is done by people who don't master them, the software crisis will remain with us and will be considered an incurable disease. And you know what incurable diseases do: they invite the quacks and charlatans in, who in this case take the form of Software Engineering Gurus."

– Edsger Dijkstra

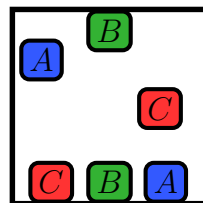
Polya's Mouse

"A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom." – Polya

Connect tiles of the same letter with wires. Wires cannot cross, enter tiles, or leave the box. How can it be done? If it can't be done, why not?

Don't be quick to dismiss either conclusion. Try this and that. Fiddle around until you understand the problem and the difficulty. Patience.

To solve such problems, "*You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.*" – Polya.

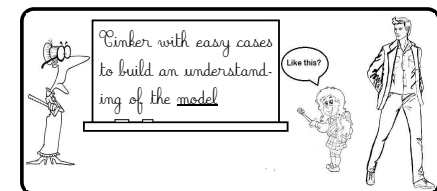
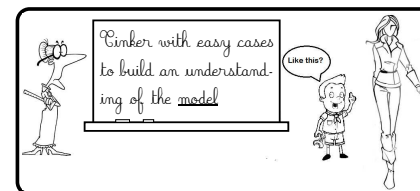


Getting Good at Discrete Math

The *professional's* workflow in addressing a discrete math problem:

- 1: Model the problem you are trying to solve using a discrete mathematical object.
- 2: Tinker with easy cases to build an understanding of the model.
- 3: Based on the tinkering, formulate a conjecture about your problem/model.
- 4: Prove the conjecture and make it a theorem. You now *know* something new.

Tinker, Tinker, Tinker, **Tinker!**



Three Challenge Problems

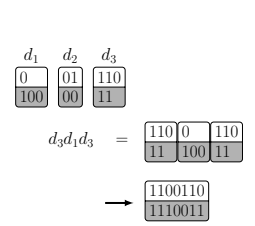
\$100

Distinct subsets with the same sum

571982530567961346538159829
 5487945882843158696672157984
 4767666531754254874224257763
 182092435975732125866239784
 428871421589197614711647671
 796713196176885488954217186
 257296727666133789225764888
 1294867141921952639693619381
 476441383328911801690185046
 147438618523176922667154474
 257864976368491316342932833
 51615898528468691977898714
 22482309898185551523961879
 7474189614567412367516833398
 621185567334949471748161445
 4942716234987219251848674
 551628435967253836539861178
 5854762719618549417768925747
 531399117196892518124735471
 475709175414131407974173535
 429288614454146728246198812
 4468463715866746258976552344
 3638617182292373168181879
 125892263729296589785418839
 448279727261797827654896937
 874985322285371162986411895
 11165945736197176608309052
 3879213273596322735993329754
 9012359131574119667168196759
 3351221818818718673691977472
 8858532281251268896449076
 4332859488712592255418653
 242871158237196443381751663
 673818968886175787842701
 8794331221317512939759215
 2989694248827479769152313629
 61174544279877311467389412
 27631844839197658442393436
 688421474699788976433695787
 86718292183175741753682814
 9431156857244768326468989597
 478448664774883858184109
 3624757247737414772711373622
 936181976428621812121963395
 98933151615642258128354434
 591392398883875289562158982
 831389154856672814692858479
 22658631851879114874613909
 347718428963434321175214
 6321349612522496241515883478

\$1,000

Domino Program



Goal: Want same top and bottom.

Domino program:

Input: dominos
 Output: sequence that works
 or
 say it can't be done

\$10

Create the best 'math'-cartoon

Create a cartoon to illustrate some discrete math you learned in this class.

If you submit one, I can use it in the future