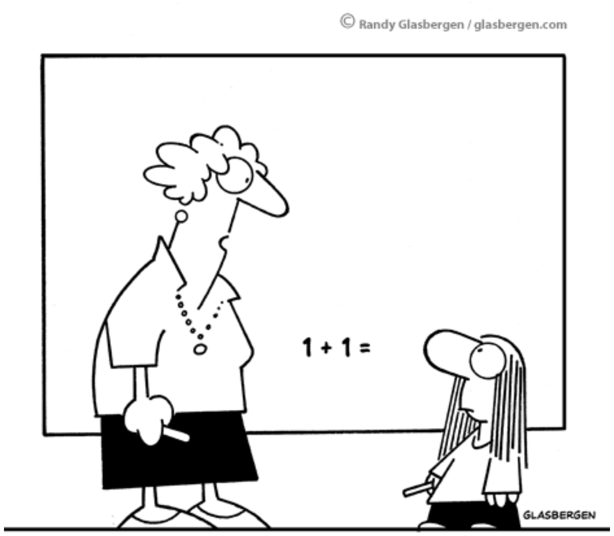
Foundations of Computer Science
Lecture 1

Warmup: A Taste for Discrete Math and Computing

Background
Disease spread, speed-dating, friendship networks
3 Challenge Problems



"Yes, this will be useful to you later in life."

"It's important to learn math because someday you might accidentally buy a phone without a calculator."

"In 1953 you were my math teacher. You promised that algebra would come in handy someday. How much longer do I have to wait?"

(Today) Warmup: A Taste for Discrete Math and Computing

Resources and Rules

1. Web Page: www.cs.rpi.edu/∼magdon/courses/focs.php
   – course info: www.cs.rpi.edu/∼magdon/courses/focs/info.pdf
   – schedule+reading+slides: www.cs.rpi.edu/∼magdon/courses/focs/slides.html
   – assignments+exams: www.cs.rpi.edu/∼magdon/courses/focs/assign.html
2. Text Book: Discrete Mathematics and Computing (Magdon-Ismail).
3. TAs, UG-Mentors.
4. Recitation Section.
5. ALAC Drop-in-tutoring.
6. Professor.
7. **Prerequisites:**
   CS II (data structures)
   Calc I (Calc II STRONGLY recommended)
8. **Rules:** No food, no electronics, no cheating.

The Storyline

1. Discrete objects.
2. Reasoning about discrete objects
3. Counting discrete objects
4. Randomness: probability
5. What can we compute?
6. What can we compute efficiently?

- concepts/concrete
- proof/theory/abstract
- theory of computation

our language will be mathematics . . .
. . . it will be everywhere

Background

Programming, numbers, geometry, algebra, calculus, . . .

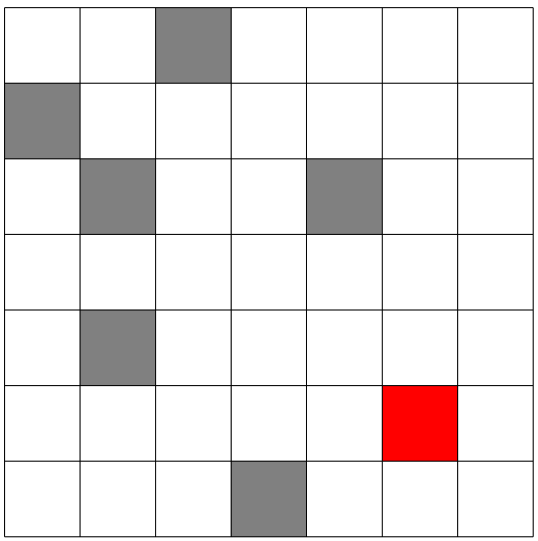
- What is the minimum element in the set $\{8, 9, 3, 10, 19\}$?
- Does this set of *positive* numbers have a minimum element:

$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

Any (non-empty) set containing only **positive integers** has a minimum element.

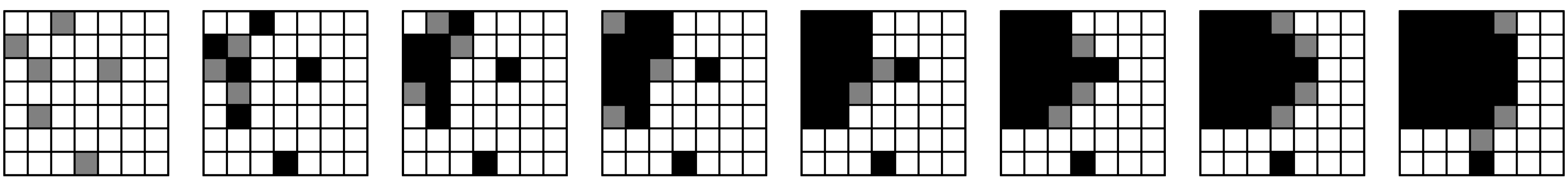
Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.



- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?
- Given few vaccines, who to immunize?
- What were the "entry points"?

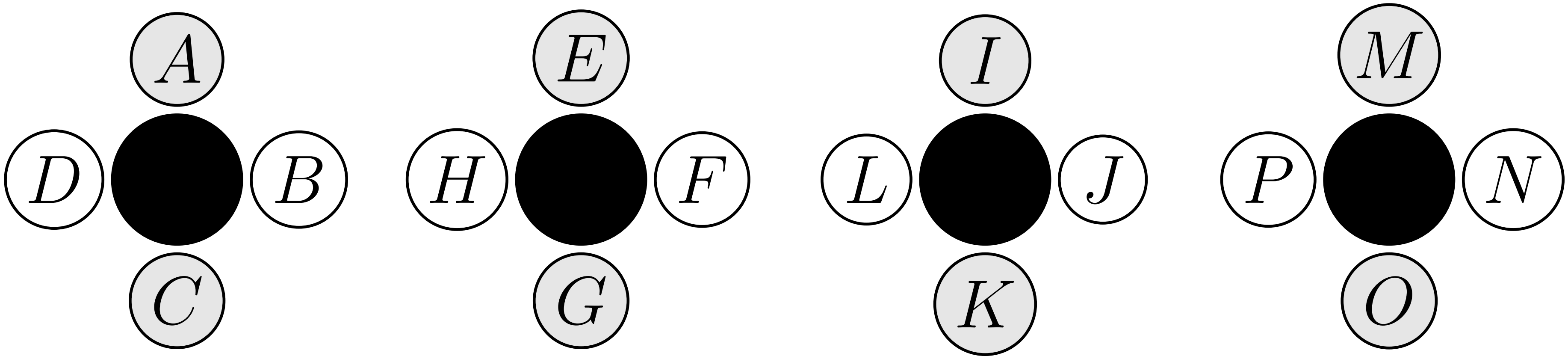
Answers involve discrete math.



day 1 day 2 day 3 day 4 day 5 day 6 day 7 day 8

Scheduling Speed Dates

In each round 4 people "group"-speed-date around a table. (4 rounds in all)



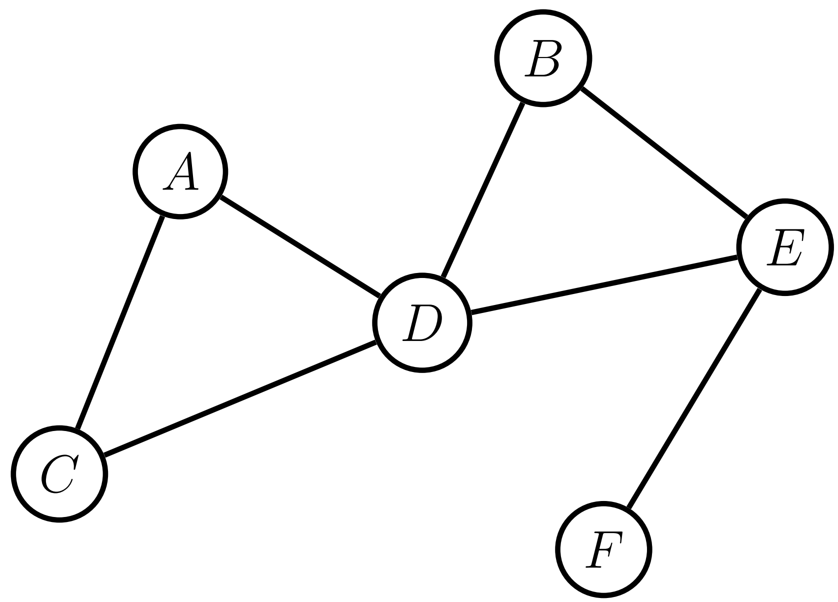
How to organize the rounds so that people meet as many people as possible?

- Do you care about average or minimum number of meetups per person?
- Can everyone meet at least 10 people?
- What happens if you assign tables randomly?

Answers involve discrete math.

Friendship Networks and Ads

People are circles and links are friendships.



Who would you advertise to? You wish to maximize adoption of your new technology.

Answers involve discrete math.

Modeling Computers

Desktop, smartphone, fitbit, ...

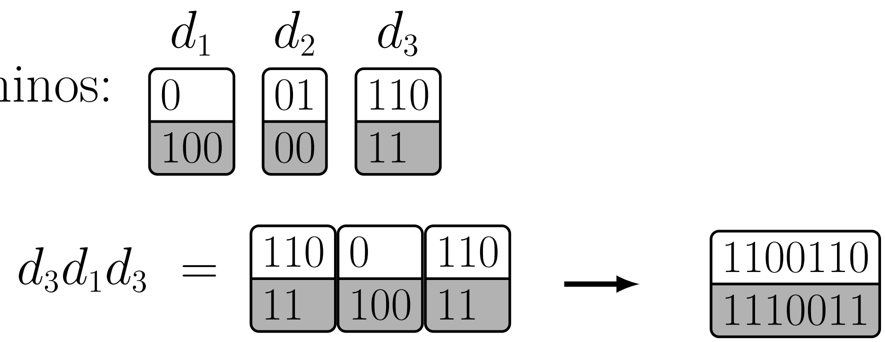
We have deep questions:

1. What can we compute?
2. What *can't* we compute?
3. How fast?

Answers involve discrete math.

What is computing?

Dominos:



Domino puzzle: Want same top and bottom.

Domino program:

Input: dominos
Output: sequence that works
or
say it can't be done

Computing is Mathematics

"Too few people recognize that the high technology so celebrated today is essentially a mathematical technology."

"A programmer must *demonstrate* that their program has the required properties. If this comes as an afterthought, it is all but certain that they won't be able to meet this obligation. Only if this obligation influences the design is there hope to meet it...

"The required techniques of effective reasoning are pretty formal, but as long as programming is done by people who don't master them, the software crisis will remain with us and will be considered an incurable disease. And you know what incurable diseases do: they invite the quacks and charlatans in, who in this case take the form of Software Engineering Gurus."

– Edsger Dijkstra

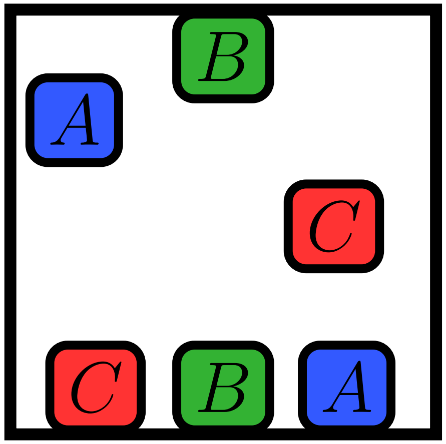
Polya's Mouse

"A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom." – Polya

Connect tiles of the same letter with wires. Wires cannot cross, enter tiles, or leave the box. How can it be done? If it can't be done, why not?

Don't be quick to dismiss either conclusion. Try this and that. Fiddle around until you understand the problem and the difficulty. Patience.

To solve such problems, "*You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.*" – Polya.

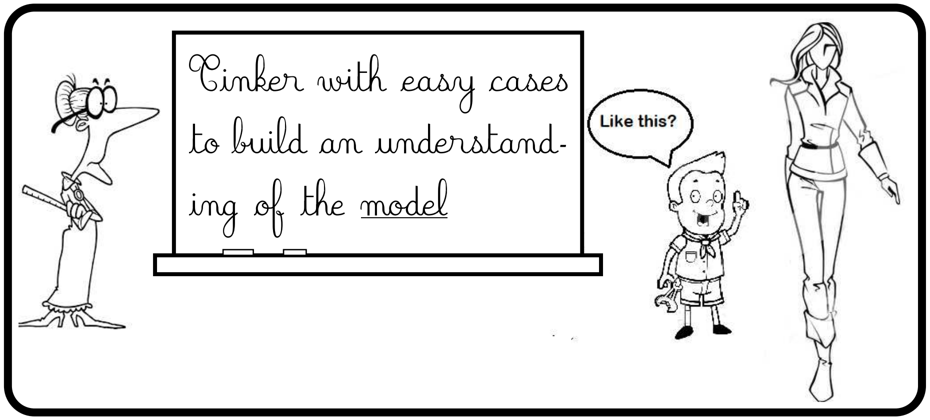


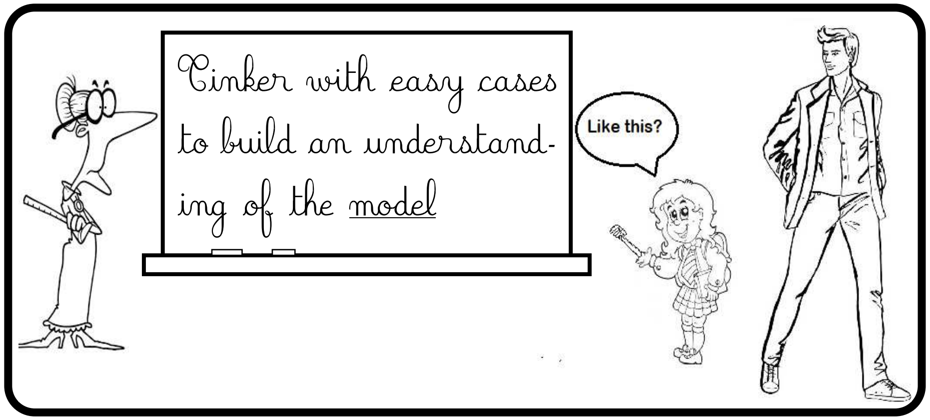
Getting Good at Discrete Math

The *professional*'s workflow in addressing a discrete math problem:

1: Model the problem your are trying to solve using a discrete mathematical object.
2: Tinker with easy cases to build an understanding of the model.
3: Based on the tinkering, formulate a conjecture about your problem/model.
4: Prove the conjecture and make it a theorem. You now *know* something new.

Tinker, Tinker, Tinker, Tinker!





Three Challenge Problems

$100

Distinct subsets with the same sum

5719825393567961346558155629	1796439694824213266958886393
5487945882843158696672157984	6366252531759955676944496585
4767766531754254874224257763	8545458545636898974365938274
1855924359757732125866239784	3362291186211522318566852576
4289776424589197647513647977	8464473866375474967347772855
7967131961768854889594217186	2892857564355262219965984217
2572967277666133789225764888	4296693937661266715382241936
1294587141921952639693619381	8634764617265724716389775433
4764413635323911361699183586	8415234243182787534123894858
1474343641823476922667154474	2267353254454872616182242154
2578649763684913163429325833	4689911847578741473186337883
5161596985226568681977938754	4428766787964834371794565542
2242632698981685551523361879	7146295186764167268433238125
7474189614567412367516833398	2273823813572968577469388278
6211855673345949471748161445	6686132721336864457635223349
4942716233498772219251848674	3161518296576488158997146221
5516264359672753836539861178	1917611425739928285147758625
5854762719618549417768925747	3516431537343387135357237754
5313691171963952518124735471	7549684656732941456945632221
6737691754241231469753717635	2397876675349971994958579984
4292388614454146728246198812	4675844257857378792991889317
4468463715866746258976552344	2832515241382937498614676246
2638621731822362373162811879	8755442772953263299368382378
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1116599457961971796683936952	1471226144331341144787865593
3879213273596322735993329751	3545439374321661651385735599
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2989694245827479769152313629	7568842562748136518615117797
6117454427987751131467589412	2776621559882146125114473423
2761854485919763568442339436	6174299197447843873145457215
6884214746997985976433695787	5387584131525787615617563371
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8313891548569672814692858479	2831727715176299968774951996
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3477184288963424358211752214	9954744594922386766735519674
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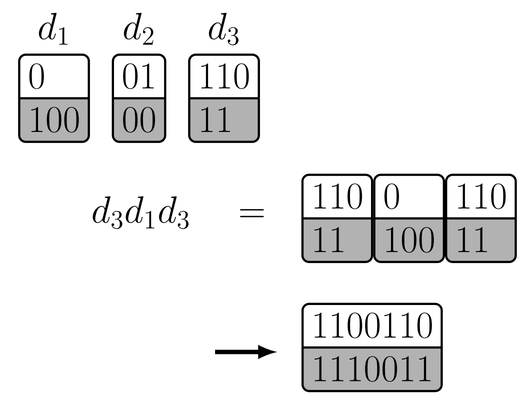
$1,000

Domino Program



Goal: Want same top and bottom.

Domino program:

Input: dominos

Output: sequence that works

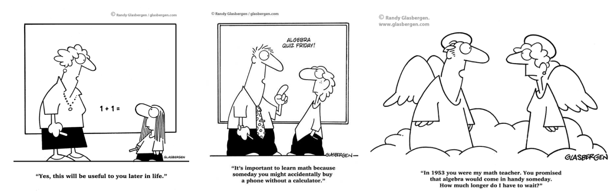
or

say it can't be done

$10

Create the best 'math'-cartoon

Create a cartoon to illustrate some discrete math you learned in this class.



If you submit one, I can use it in the future