Learning From Data
Lecture 2
The Perceptron

The Learning Setup
A Simple Learning Algorithm: PLA
Other Views of Learning
Is Learning Feasible: A Puzzle

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RECAP: # The Plan

1. What is Learning?
2. Can We do it?
3. How to do it?
4. How to do it well?
5. General principles?
6. Advanced techniques.
7. Other Learning Paradigms.

- concepts
- theory
- practice

our language will be mathematics ...
...our sword will be computer algorithms

RECAP: The Key Players

- Salary, debt, years in residence, ...
- Approve credit or not
- True relationship between $\mathbf{x}$ and y
- Data on customers

input $\mathbf{x} \in \mathbb{R}^d = \mathcal{X}$.

output $y \in \{-1, +1\} = \mathcal{Y}$.

target function $f : \mathcal{X} \mapsto \mathcal{Y}$.

(The target f is *unknown*.)

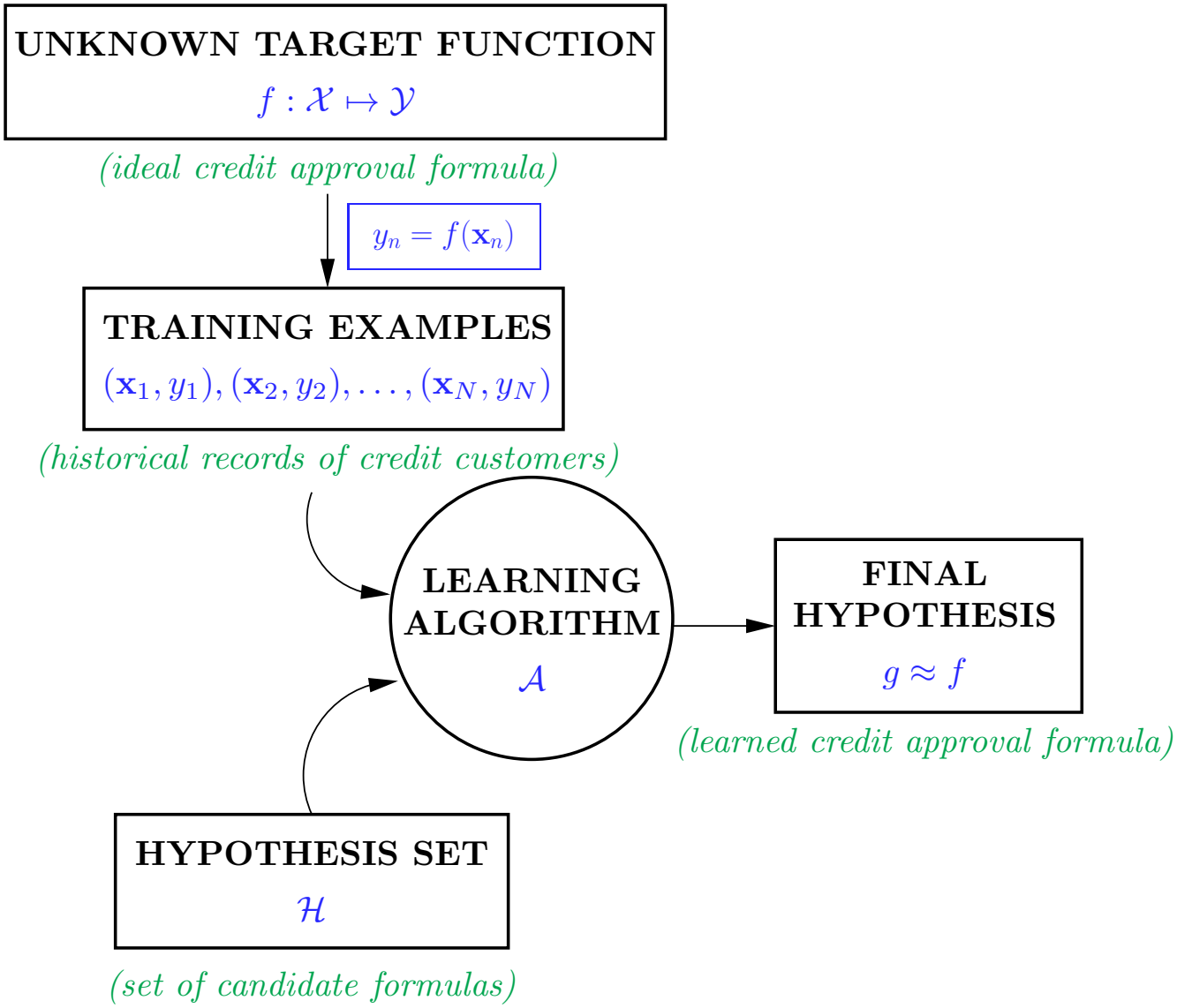
data set $\mathcal{D} = (\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N)$.

($y_n = f(\mathbf{x}_n)$.)

$\mathcal{X}$ $\mathcal{Y}$ and $\mathcal{D}$ are *given* by the learning problem;
The target f is fixed but unknown.

We learn the function f from the data $\mathcal{D}$.

RECAP: Summary of the Learning Setup



A Simple Learning Model

- Input vector $\mathbf{x} = [x_1, \ldots, x_d]^{\mathrm{T}}$.
- Give importance weights to the different inputs and compute a "Credit Score"

$$\text{"Credit Score"} = \sum_{i=1}^{d} w_i x_i.$$

- Approve credit if the "Credit Score" is acceptable.

$$\text{Approve credit if } \sum_{i=1}^{d} w_i x_i > \text{threshold}, \quad \text{("Credit Score" is good)}$$

$$\text{Deny credit if } \sum_{i=1}^{d} w_i x_i < \text{threshold}. \quad \text{("Credit Score" is bad)}$$

- How to choose the importance weights w_i

input x_i is important	$\Longrightarrow$	large weight $\lvert w_i \rvert$
input x_i beneficial for credit	$\Longrightarrow$	positive weight $w_i > 0$
input x_i detrimental for credit	$\Longrightarrow$	negative weight $w_i < 0$

A Simple Learning Model

$$\text{Approve credit if } \sum_{i=1}^{d} w_i x_i > \text{threshold},$$

$$\text{Deny credit if } \sum_{i=1}^{d} w_i x_i < \text{threshold}.$$

can be written formally as

$$h(\mathbf{x}) = \text{sign}\left(\left(\sum_{i=1}^{d} w_i x_i\right) + w_0\right)$$

The "bias weight" w_0 corresponds to the threshold. (How?)

The Perceptron Hypothesis Set

We have defined a *Hyopthesis set* $\mathcal{H}$

$$\mathcal{H} = \{h(\mathbf{x}) = \text{sign}(\mathbf{w}^{\text{T}}\mathbf{x})\} \qquad \longleftarrow \text{uncountably infinite } \mathcal{H}$$

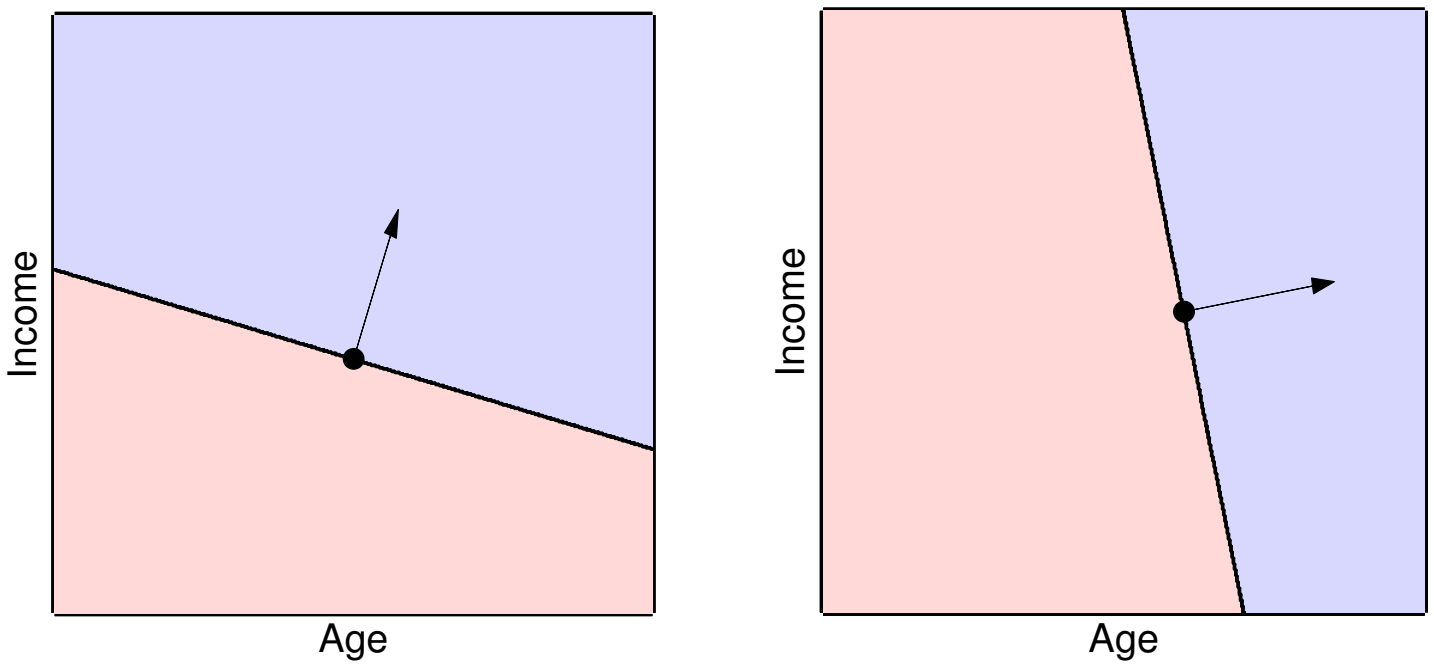
$$\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_d \end{bmatrix} \in \mathbb{R}^{d+1}, \qquad \mathbf{x} = \begin{bmatrix} 1 \\ x_1 \\ \vdots \\ x_d \end{bmatrix} \in \{1\} \times \mathbb{R}^d.$$

This hypothesis set is called the *perceptron* or *linear separator*

Geometry of The Perceptron

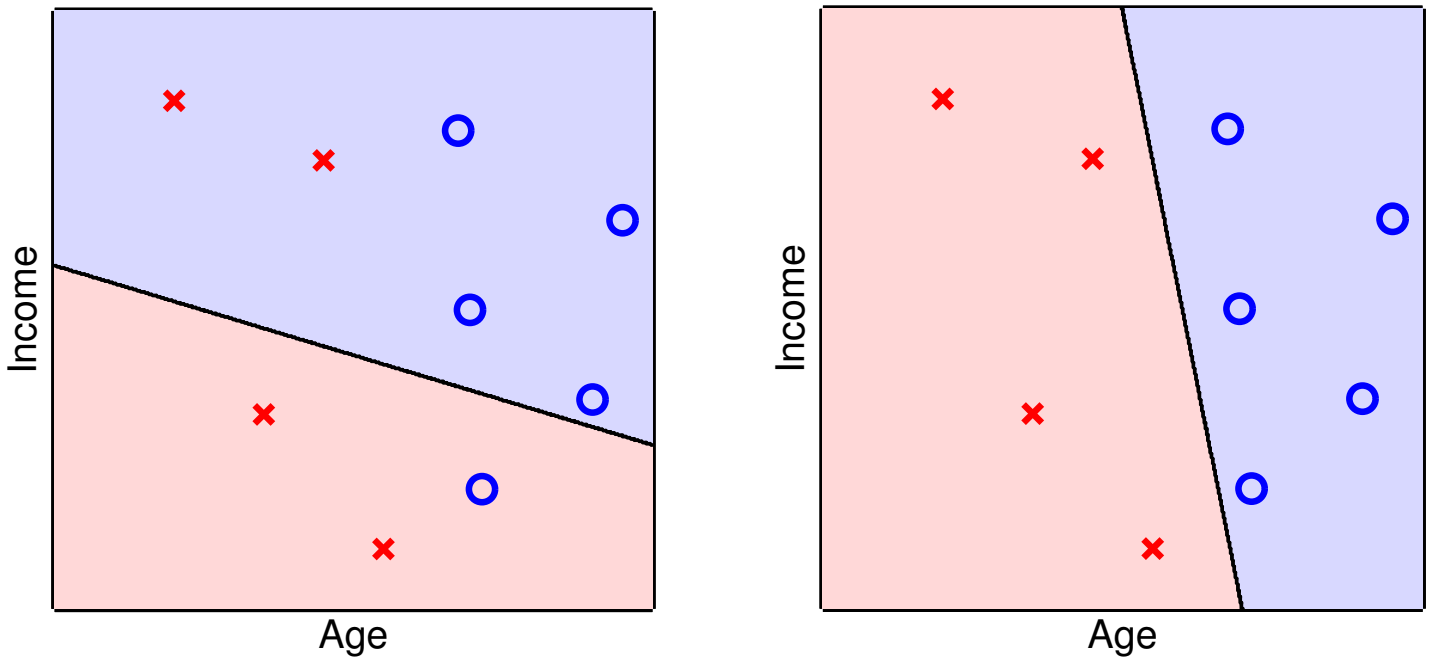
$$h(\mathbf{x}) = \text{sign}(\mathbf{w}^{\text{T}}\mathbf{x})$$

(Problem 1.2 in LFD)



Which one should we pick?

Use the Data to Pick a Line



A perceptron fits the data by using a line to separate the $+1$ from -1 data.

Fitting the data: How to find a hyperplane that *separates* the data?

("It's obvious - just look at the data and draw the line," is not a valid solution.)

How to *Learn* a Final Hypothesis g from $\mathcal{H}$

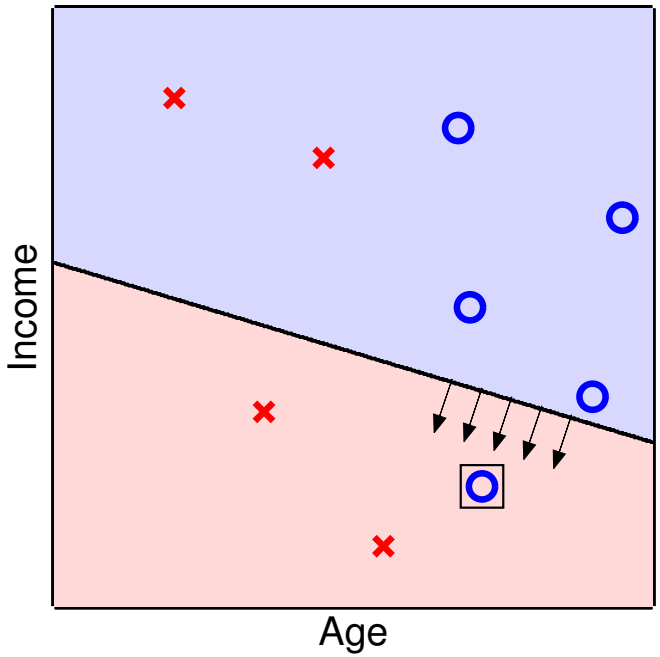
We want to select $g \in \mathcal{H}$ so that $g \approx f$.

We certainly want $g \approx f$ on the data set $\mathcal{D}$. Ideally,

$$g(\mathbf{x}_n) = y_n.$$

How do we find such a g in the *infinite* hypothesis set $\mathcal{H}$, if it exists?

Idea! Start with some weight vector and try to improve it.



The Perceptron Learning Algorithm (PLA)

A simple iterative method.

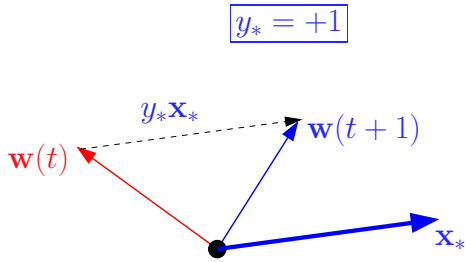
1: $\mathbf{w}(1) = \mathbf{0}$
2: **for** iteration $t = 1, 2, 3, \ldots$
3: the weight vector is $\mathbf{w}(t)$.
4: From $(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_N, y_N)$ pick any misclassified example.
5: Call the misclassified example $(\mathbf{x}_*, y_*)$,

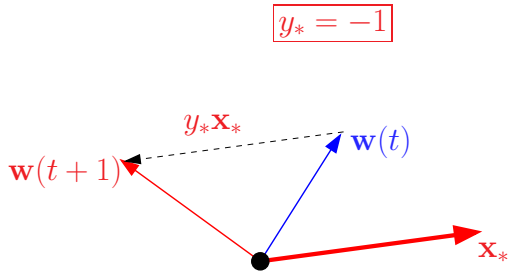
$$\text{sign}\left(\mathbf{w}(t) \bullet \mathbf{x}_*\right) \neq y_*.$$

6: Update the weight:

$$\mathbf{w}(t+1) = \mathbf{w}(t) + y_*\mathbf{x}_*.$$

7: $t \leftarrow t + 1$



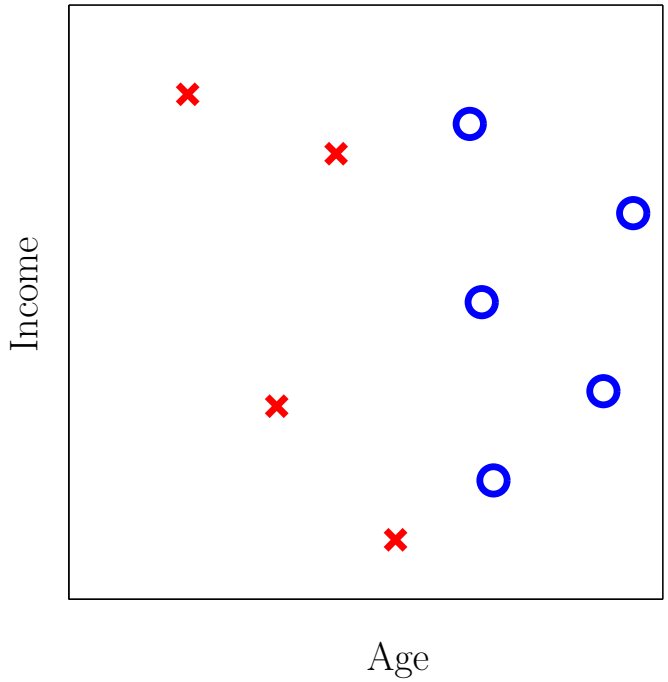


PLA implements our idea: start at some weights and try to improve.

"incremental learning"on a single example at a time

Does PLA Work?

Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.



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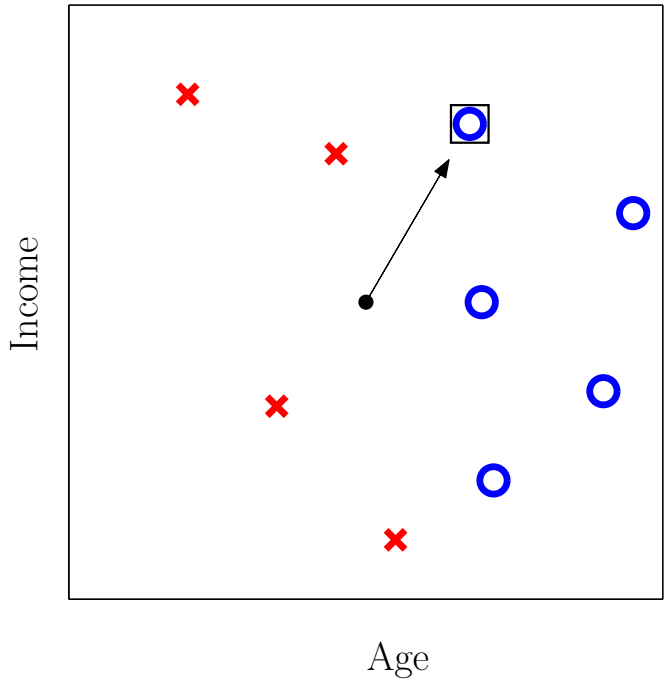

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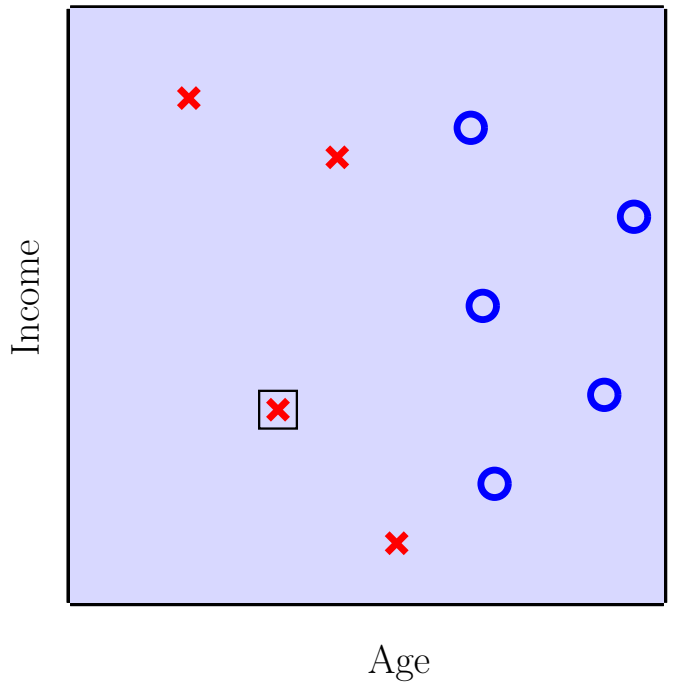
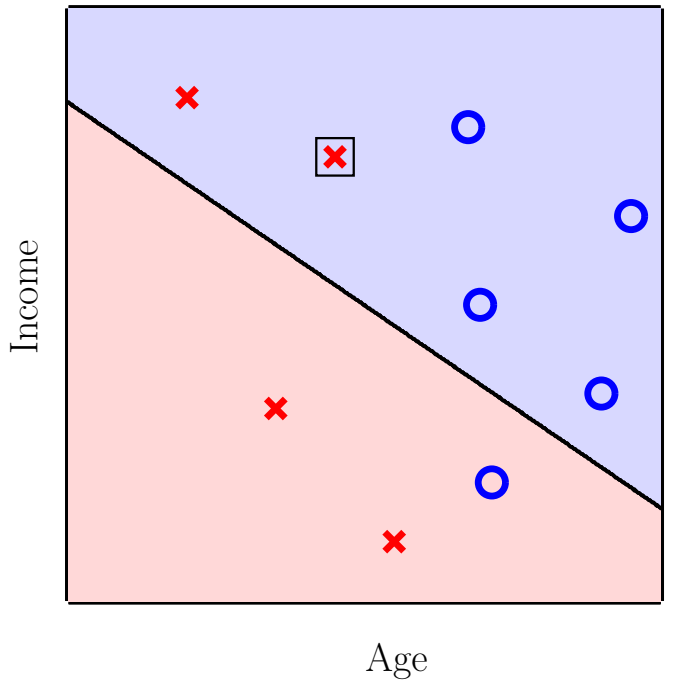


ITERATION 1

Does PLA Work?

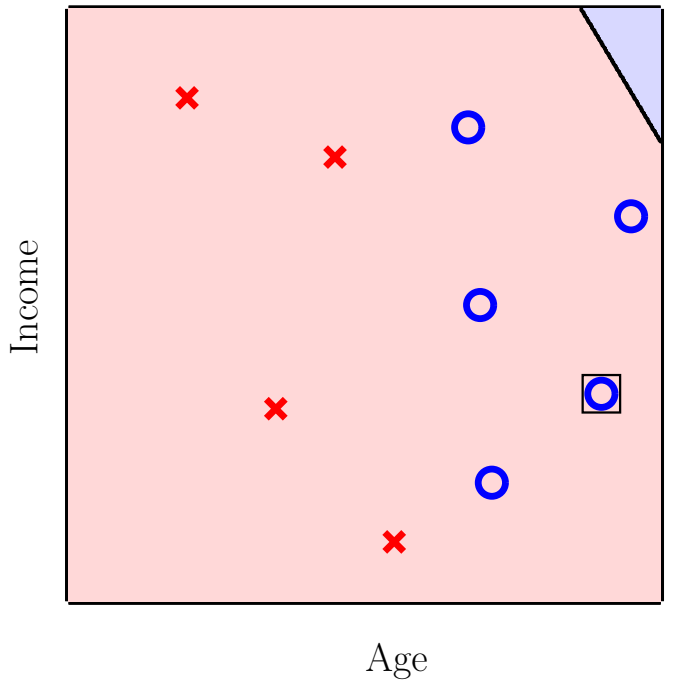
Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.



ITERATION 2

Does PLA Work?

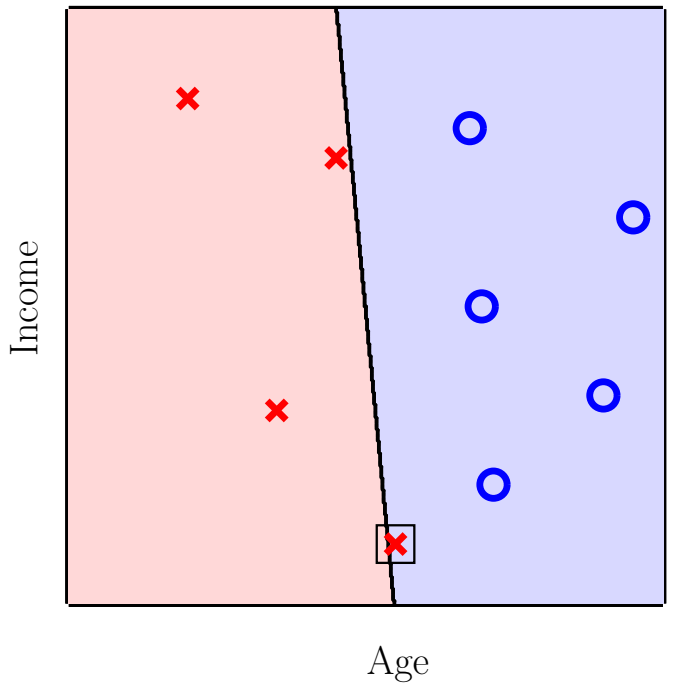
Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.



ITERATION 3

Does PLA Work?

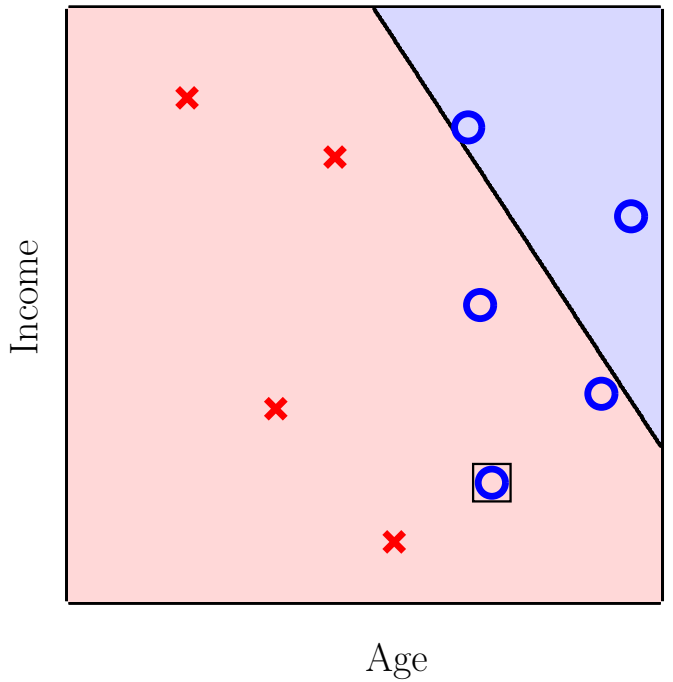
Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.



ITERATION 4

Does PLA Work?

Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.

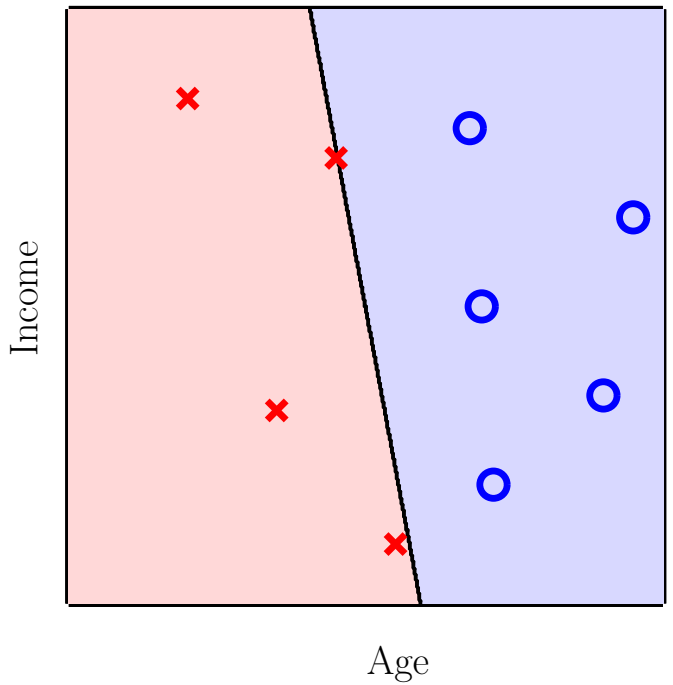


ITERATION 5

Does PLA Work?

Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.

After how long?



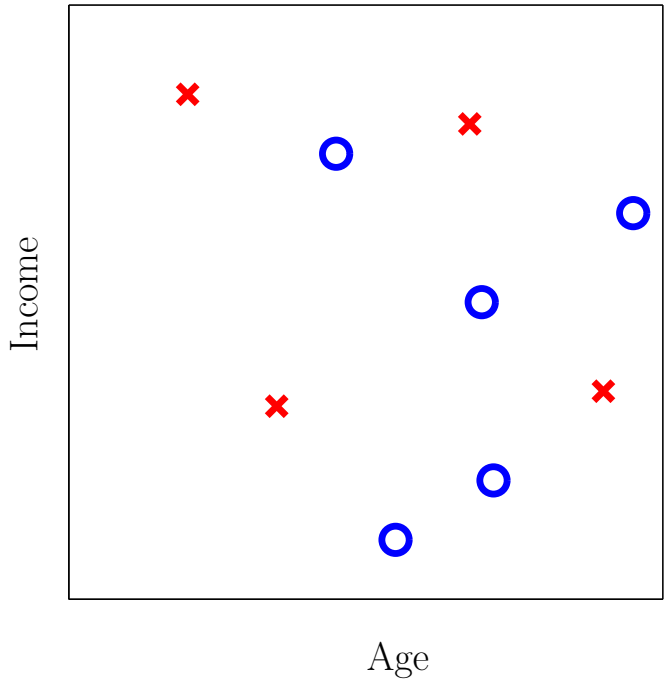
ITERATION 6

Does PLA Work?

Theorem. If the data can be fit by a linear separator, then after some *finite* number of steps, PLA will find one.

After how long?

What if the data cannot be fit by a perceptron?



We can Fit the Data

- We can find an h that works from infinitely many (for the perceptron).
  (So computationally, things seem good.)

- Ultimately, remember that we want to *predict*.
  We don't care about the data, we care about *"outside the data"*.

Can a limited data set reveal enough information to pin down an entire target function, so that we can predict outside the data?

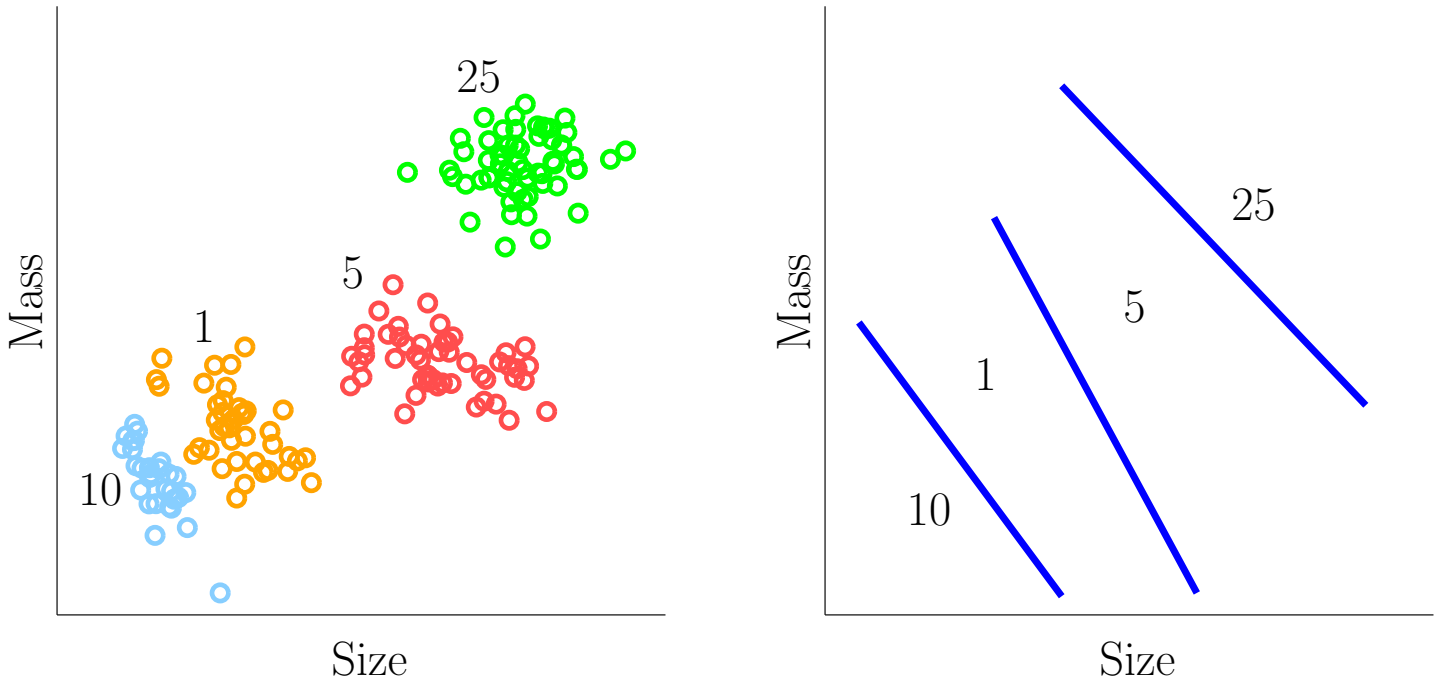
Other Views of Learning

- Design: learning is from data, design is from specs and a model.
- Statistics, Function Approximation.
- Data Mining: find patterns in massive data (typically unsupervised).
- Three Learning Paradigms
  - Supervised: the data is $(\mathbf{x}_n, f(\mathbf{x}_n))$ – you are told the answer.
  - Reinforcement: you get feedback on potential answers you try:

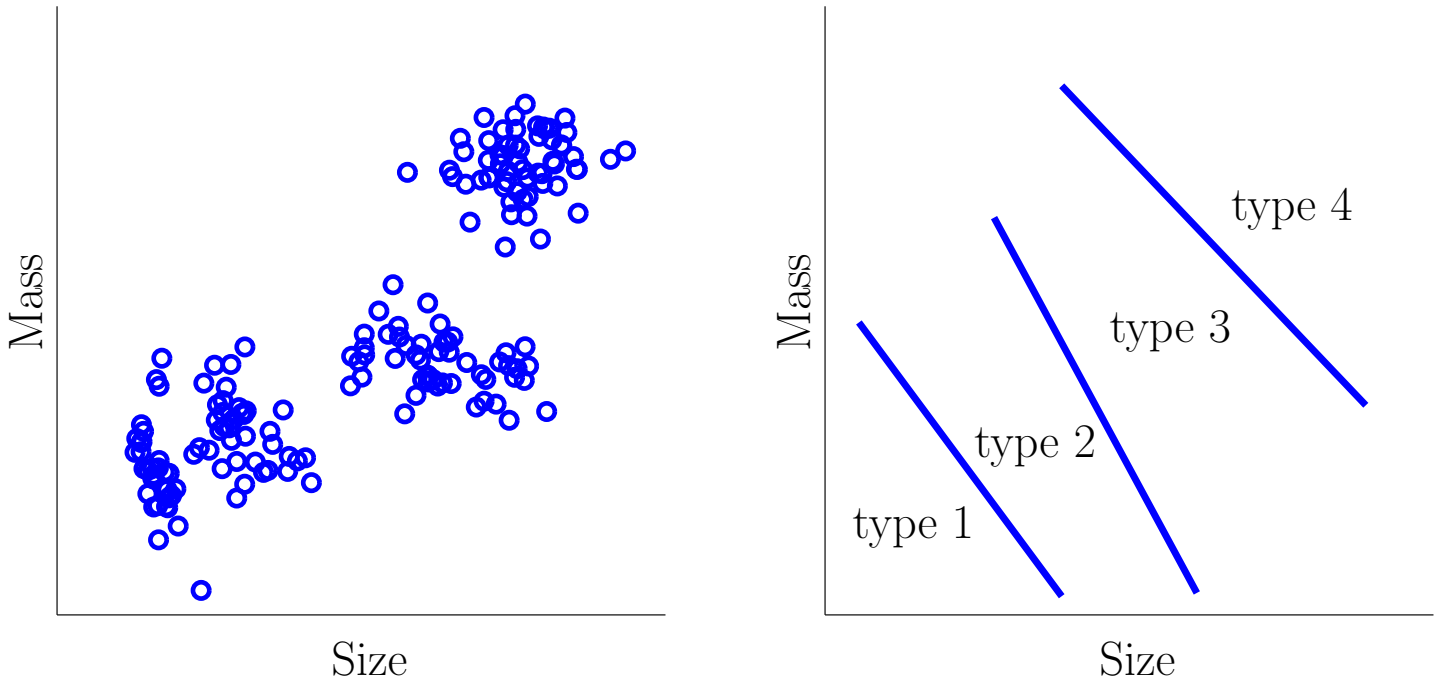
    $$\mathbf{x} \rightarrow \text{ try something } \rightarrow \text{ get feedback.}$$

  - Unsupervised: only given $\mathbf{x}_n$, learn to "organize" the data.

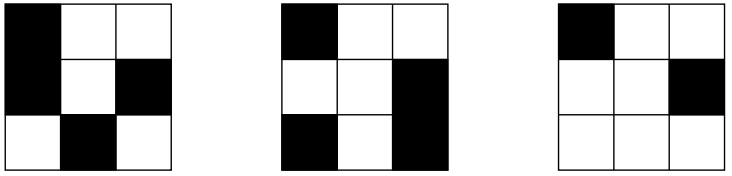
Supervised Learning - Classifying Coins



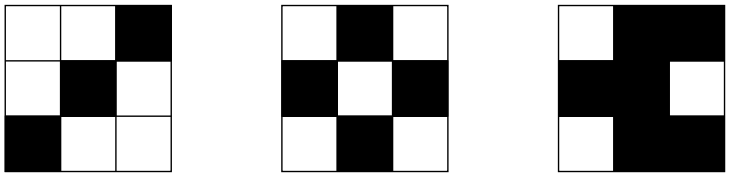
Unsupervised Learning - Categorizing Coins



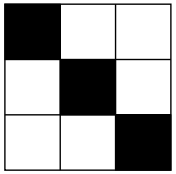
Outside the Data Set - A Puzzle



Dogs ($f = -1$)



Trees ($f = +1$)



Tree or Dog? ($f = ?$)