FINAL: 180 Minutes

Last Name: ____________________

First Name: ____________________

RIN: ____________________

Section: ____________________

Answer **ALL** questions. You may use **two** double sided $8\frac{1}{2} \times 11$ crib sheets.

You **MUST** show **CORRECT** work (even for multiple choice) to receive full credit.

NO COLLABORATION or electronic devices. Any violations result in an F.

NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

1	2	3	4	5	6	Total
200	30	30	30	30	30	350

1 Circle at most one answer per question. 10 points for each correct answer.

(1) Is this claim true or false. $\forall n \in \mathbb{Z} : n^2 \geq 0$.

A True.

B False.

C You can't say because it depends on n.

D You can't assign true or false to quantified statements.

E It is not a proper statement to which you can assign true or false.

(2) If it rains on a day, it must rain the next day. Today it did not rain. What can you conclude?

A It won't rain tomorrow.

B It won't rain on any future day.

C It rained yesterday.

D It did not rain yesterday but it could have rained on some day prior to yesterday.

E It did not rain yesterday and it did not rain on any day prior to yesterday.

(3) To prove $P(n)$ by induction, which is *not* a valid induction step to prove $P(n) \to P(n+1)$.

A *Assume* that $P(n)$ is true and *prove* that $P(n+1)$ is true.

B *Assume* two things, that $P(n)$ is true and that $P(n+1)$ is false. Now derive a contradiction.

C *Assume* that $P(n)$ is false and *prove* that $P(n+1)$ is false.

D *Assume* that $P(n+1)$ is false and *prove* that $P(n)$ is false.

E All of the above are valid induction steps.

(4) What is the approximate value of the sum $\sum_{i=0}^{20} (2^i + i)(2^i - i)$.

A 1.5×10^{11}.

B 4.0×10^{11}.

C 1.5×10^{12}.

D 4.0×10^{12}.

E 1.5×10^{13}.

(5) $T_1 = 1$ and $T_n = T_{n-1} + n^2$ for $n > 1$. Which order relationship is accurate?

A $T_n \in \Theta(n)$.

B $T_n \in \Theta(n^2)$.

C $T_n \in \Theta(n^3)$.

D $T_n \in \Theta(2^n)$.

E None of the above.

(6) What is the remainder when 2^{2019} is divided by 5?

A 0.

B 1.

C 2.

D 3.

E 4.

(7) Define the set $A = \{3x + 7y \mid x \text{ and } y \text{ are in } \mathbb{Z}\}$. Which numbers are *not* in A?

A -11.

B 11.

C 37.

D 142.

E They are all in A.

(8) Ayfos is in a social network with 14 others, so 15 people in all with Ayfos. There are 25 friendship links in this network. Everyone but Ayfos has 3 friends. How many friends does Ayfos have?

A 6.

B 7.

C 8.

D 9.

E Can't be determined or such a social network cannot exist.

(9) In the previous problem regarding Ayfos' social network, you pick a person randomly. What is the expected number of friends that person has.

A $3\frac{1}{3}$.

B $3\frac{1}{2}$.

C $3\frac{3}{4}$.

D 4.

E None of the above, or not enough information to say for sure.

(10) From 1000 students, 900 are CS and 200 are MATH. How many are CS-MATH duals?

A 50.

B 100.

C 150.

D 200.

E None of the above, or not enough information to say for sure.

(11) Digits are 0,1,. . . , 9. How many of the three digit strings 000 to 999 have a digit-sum 10? (For example, 307 and 811 have digit sum 10, but 846 and 213 do not.)

A 60.

B 63.

C 66.

D 69.

E None of the above.

(12) A and B are sets. $|A| = 5$ and $|B| = 3$. How many functions are there from A to B?

A 3^5.

B 5^3.

C $5!$.

D $\binom{5}{3}$.

E None of the above.

(13) A and B are sets. $|A| = 5$ and $|B| = 3$. How many injections (1-to-1) are there from A to B?

A 0.

B 100.

C 150.

D 200.

E None of the above.

(14) A and B are sets. $|A| = 5$ and $|B| = 3$. How many surjections (onto) are there from A to B?

A 0.

B 100.

C 150.

D 200.

E None of the above.

(15) You roll a die 4 times. What is the probability to get (exactly) 2 sixes?

A $6/6^4$.

B $12/6^4$.

C $36/6^4$.

D $150/6^4$.

E None of the above.

(16) Al and Jo each independently pick 4 restaurants randomly from 10 restaurants $r_1, \ldots, r_{10}$. They must eat at a restaurant that both picked. Compute the probability they can eat at (exactly) 2 restaurants.

A 2/7

B 3/7

C 4/7

D 5/7

E None of the above

(17) Compute the expected number of restaurants Al and Jo from the previous problem can eat at.

A 1.2.

B 1.4

C 1.6.

D 1.8

E None of the above

(18) Which computing problem *cannot* be solved by a DFA?

A Strings with an even number of 1s.

B Strings which have more 1s than 0s.

C Strings whose number of 1s is a multiple of 3.

D Strings whose number of 1s is not a multiple of 3.

E Each problem is solvable using a DFA

(19) Which string cannot be generated by the CFG $S \rightarrow \varepsilon|0S|1S$?

A 11111111110000000000 $= 1^{\bullet 10}0^{\bullet 10}$.

B 10101010101010101010 $= (10)^{\bullet 10}$.

C 00000000000000000000 $= 0^{\bullet 20}$.

D 00110011001100110011 $= (0011)^{\bullet 5}$.

E They can all be generated.

(20) Which answer is a valid conclusion about the decidability of the language $\mathcal{L}_B$?

A $\mathcal{L}_A$ is decidable. A decider for $\mathcal{L}_B$ can be converted to a decider for $\mathcal{L}_A$. So, $\mathcal{L}_B$ is decidable.

B $\mathcal{L}_A$ is decidable. A decider for $\mathcal{L}_A$ can be converted to a decider for $\mathcal{L}_B$. So, $\mathcal{L}_B$ is decidable.

C $\mathcal{L}_A$ is undecidable. A decider for $\mathcal{L}_A$ can be converted to a decider for $\mathcal{L}_B$. So, $\mathcal{L}_B$ is undecidable.

D $\mathcal{L}_A$ is undecidable. A decider for $\mathcal{L}_B$ can be converted to a decider for $\mathcal{L}_A$. So, $\mathcal{L}_B$ is decidable.

E None of the above is valid.

2 Determine the Type of Proof and Prove

Prove that for $n \in \mathbb{N}$, $\sqrt{n(n+1)} \leq n + \frac{1}{2}$.

3 Induction and Sums. Tinker, Tinker, Tinker.

For $n \in \mathbb{N}$, obtain a formula for the sum $S(n) = \sum_{i=1}^{2n} (-1)^i i$ and prove your formula by induction.

4 Expected Waiting Time to 3 Heads In A Row

You flip a fair coin until you get 3 heads *in a row*. Compute the expected number of flips you make.

5 CFGs and Induction. (Tinker, tinker,. . .)

For the CFG $S \to 0|0S1$, *prove* that every string that can be generated has odd length.

6 Turing Machine for Squaring.

Give a high level pseudo-code description of a Turing Machine that solves the problem $\mathcal{L} = \{0^{\bullet n}1^{\bullet n \times n} | n \geq 1\}$. (You do not need to give machine level details but your pseudo-code should demonstrate understanding of how the Turing Machine moves back and forth to solve the problem. Tinker.)

SCRATCH

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