MIDTERM: 90 Minutes

Last Name: ____________________

First Name: ____________________

RIN: ____________________

Section: ____________________

Answer **ALL** questions. You may use a single sided $8\frac{1}{2} \times 11$ crib sheet.

NO COLLABORATION or electronic devices. Any violations result in an F.

NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

1	2	3	4	5	Total

50	50	50	50	50	250

1 Circle at most one answer per question. 10 points for each correct answer and -5 points for each incorrect answer (blank answer is 0 points). Don't guess!

(a) $p \to (q \wedge r)$ is equivalent to what other compound proposition:

[A] $(p \to q) \wedge r$

[B] $(p \to q) \wedge (p \to r)$

[C] $(p \wedge q) \to r$

[D] $p \vee (q \wedge r)$

(b) The **negation** of "If Malik is in pajamas then all lights are off" is

[A] Malik is in pajamas and at least one light is on

[B] Malik is in pajamas or all lights are off

[C] Malik is not in pajamas and at least one light is on

[D] Malik is not in pajamas and all lights are off

(c) $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \to (P(n^2) \wedge P(n-2))$ is true for $n \geq 2$. For which n can we be **sure** $P(n)$ is true?

[A] All $n \geq 2$.

[B] All even $n \geq 0$.

[C] All odd $n \geq 0$.

[D] All n which are perfect squares.

(d) Compute the remainder when 2014^{2014} is divided by 5? *[Hint:* $2014 \equiv -1 \pmod 5$.*]*

[A] $r = 1$

[B] $r = 2$

[C] $r = 3$

[D] $r = 7$

(e) A friendship network has 7 people and each person has 5 friends. How many edges (friendship links) are there in this friendship network?

[A] 17 edges

[B] 18 edges

[C] Not enough information to determine the number of edges

[D] This friendship network cannot possibly exist

2 Induction Proofs

1. Prove by <u>induction</u> that for all integer $n \geq 1$: $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = 1 - \frac{1}{n+1}$.

2. Suppose $a \equiv b \pmod{k}$. Prove by <u>induction</u> that for all integer $n \geq 1$: $a^n \equiv b^n \pmod{k}$.
   ($x \equiv y \pmod{z}$ means $x - y$ is divisible by z.)

3 Well formed arithmetic expressions

Define a set $\mathcal{A}$ of well formed arithmetic strings (sequences) with alphabet $\Sigma = \{1, +, \times, (,)\}$.

[Base Case] $1 \in \mathcal{A}$;

[Recursive Rules]
1. $x, y, z \in \mathcal{A} \rightarrow (x + y + z) \in \mathcal{A}$
2. $x, y \in \mathcal{A} \rightarrow (x \times y) \in \mathcal{A}$.

(a) Of the following three strings, circle the one that is in $\mathcal{A}$.

$$(1+1+1) \times (1+1) \qquad (1+1+1) \times ((1+1+1)+1+1) \qquad ((1+1+1) \times ((1+1+1)+1+1))$$

(b) Give a derivation of the string in (a) that is in $\mathcal{A}$. (A derivation is a sequence of strings in $\mathcal{A}$ where each string is obtained from the previous strings by applying one of the recursive rules.)

4 Evaluating arithmetic expression strings

The function eval takes an input string from the set $\mathcal{A}$ of well formed arithmetic expressions ($\mathcal{A}$ was defined in problem 3) and computes its value as an arithmetic expression. For example,

$$\mathsf{eval} : ((1+1+1) \times (((1+1+1) \times (1+1+1)) + 1 + 1)) \mapsto 33$$

<u>**Prove**</u> that for every string $x \in \mathcal{A}$, $\mathsf{eval}(x)$ is <u>**odd**</u>.

Is $((1+1+1) \times ((1+1+1)+1+1+1))$ in $\mathcal{A}$? If yes, give a <u>*derivation*</u>. If no, <u>*prove*</u> it.

5 Rooted binary trees (NOT rooted full binary trees)

Give the **recursive definition of rooted binary trees**. Explicitly state your base case and recursive rules.

Let F be the number of full nodes (with 2 children) and L the number of leaf nodes (with no children). For any non-empty rooted binary tree, prove that

$$L = F + 1.$$

SCRATCH