MIDTERM: 90 Minutes

Last Name: _____

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

1	2	3	4	5	Total
50	50	50	50	50	250

1 Circle one answer per question. 10 points for each correct answer.

(a) Compute the sum $\sum_{i=0}^{10}(i+1+2^{i+1})$

A 4150.

B 5160.

C 4149.

D 4160.

(b) Give a formula for the sum $S(n) = \sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)$

A $S(n) = n^3$

B $S(n) = n^2(n+1)$

C $S(n) = \frac{1}{2}n(n+1)$

D $S(n) = i^3 + j^3$

(c) Let $S(n) = \sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)$. Then,

A $S(n) \in \Theta(n^3)$

B $S(n) \in \Theta(n^2 \log n)$

C $S(n) \in \Theta(n)$

D $S(n) \in \Theta(n^4)$

(d) Compute the remainder when 2015^{2015} is divided by 3? *[Hint:* $2015 \equiv -1 \pmod 3$*.]*

A $r = 1$

B $r = 2$

C $r = 3$

D $r = 7$

(e) A friendship network has 7 people and each person has 4 friends. How many edges (friendship links) are there in this friendship network?

A 14 edges

B 15 edges

C Not enough information to determine the number of edges

D This friendship network cannot possibly exist

2 Computing and Proving a Sum

Give a formula for the sum $S(n) = \sum_{i=1}^{2n} (-1)^i i$. **Prove** your answer.

3 Greatest Common Divisor (GCD)

For an integer d and integers m, n, suppose that $\gcd(d, m) = 1$ and $d|mn$. **Prove** that $d|n$.

4 Regular Graphs

A graph is r-regular if every node has degree r. Let n be the number of nodes in the graph.

(a) **Prove** that if n and r are both odd, then there is no r-regular graph on n nodes.

(b) Draw examples of r-regular graphs in these two cases:
(i) $n = 7;\ r = 4$ (ii) $n = 6;\ r = 3$

5 Rooted Full Binary Trees

Give the **recursive definition of rooted full binary trees.** (State your base cases and constructor rules.)

Prove that the number of nodes in any rooted full binary tree is odd.

SCRATCH