MIDTERM: 90 Minutes

Last Name: ____________________

First Name: ____________________

RIN: ____________________

Section: ____________________

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

1	2	3	4	5	Total

100	40	40	40	40	250

(10 bonus points)

1 Circle one answer per question. 10 points for each correct answer.

(a) Which theorem below is true (n is a natural number, r is a real)?

A n^2 is divisible by 4 if and only if n is divisible by 4.

B If n^2 is divisible by 4 then n is divisible by 4.

C $n^3 + 5$ is odd if and only if n is even.

D If $\sqrt{r}$ is irrational then r is irrational.

(b) How many subsets of $\{a, b, c, d, e, f, g\}$ contain a or g?

A 90

B 92

C 94

D 96

(c) *Estimate* the value of the sum $\sum_{i=0}^{10}\sum_{j=0}^{20} 2^{i+j}$.

A 4×10^6.

B 4×10^9.

C 4×10^{12}.

D 4×10^{15}.

(d) Which is the correct asymptotic order relationship between 2^{n+1} and 2^{2n}

A $2^{n+1} \in \Theta(2^{2n})$.

B $2^{n+1} \in \omega(2^{2n})$.

C $2^{n+1} \in o(2^{2n})$.

D None of the above.

(e) Compute the sum $S = \sum_{i=1}^{3}\sum_{j=1}^{3}(i + j)$

A $S = 30$.

B $S = 32$.

C $S = 34$.

D $S = 36$.

(f) What is the last digit of n (i.e. remainder when divided by 10), where $n = 3^{2016} + 4^{2016} + 7^{2016}$?

A $r = 1$

B $r = 3$

C $r = 6$

D $r = 8$

(g) A friendship network has 5 people. The degrees (number of friends) of the people are 1,1,2,2,3. How many edges (friendship links) are in this friendship network?

A 4 edges

B 5 edges

C Not enough information to determine the number of edges

D This friendship network cannot possibly exist

(h) A friendship network has 5 people. The degrees (number of friends) of the people are 0,1,2,3,4. How many edges (friendship links) are in this friendship network?

A 4 edges

B 5 edges

C Not enough information to determine the number of edges

D This friendship network cannot possibly exist

(i) Here is some information about ice-skate options:

Colors:	White, Beige, Pink, Yellow, Blue
Sizes:	4,5,6,7,8
Extras:	Tassels, Stripes, Bells

Skates can have any combination of extras (including none). An example skate is (pink; size 5; with stripes and bells). How many types of skates are there?

A 150.

B 175.

C 200.

D 225.

(j) In an NBA game, 8 players suit up for a game. At any time only 5 players can be on the floor, playing the game. Using the 8 players, in how many ways can the 5 players on the floor be chosen?

A 56.

B 57.

C 58.

D 59.

2 Modular square-root

Let p be prime. **Prove:** $x^2 \equiv y^2 \pmod{p}$ IF AND ONLY IF $x \equiv y \pmod{p}$ or $x \equiv -y \pmod{p}$.
(If your proof uses facts from class or the book, explicitly state them as "from class" or "from the book".)

3 Playing with postage

(a) **Prove:** Any postage greater than 7¢ can be made using 3¢ and 5¢ stamps.

(b) **Prove:** For any $k \geq 1$, there is a postage $n \geq k$ that *cannot* be made using 4¢ and 6¢ stamps.

4 GCD of consecutive Fibononacci numbers

Let F_n be the nth Fibonacci number: $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$.
Prove: $\gcd(F_n, F_{n+1}) = 1$ for $n \geq 1$. (Any two consecutive Fibonacci numbers are relatively prime.)

5 Recursively defined strings

Define the set of strings $\mathcal{P}$ recursively as follows (The minimality clause is there by default.).

(1) $0, 1 \in \mathcal{P}$.

(2) There are two constructor rules: $x \in \mathcal{P} \rightarrow x\,0\,x \in \mathcal{P}$;
$x \in \mathcal{P} \rightarrow x\,1\,x \in \mathcal{P}$.

Prove that every string in $\mathcal{P}$ is a palindrome (a string that equals its reversal) and has odd length.

SCRATCH

SCRATCH