



**1 Circle one answer per question. 10 points for each correct answer.**

(1) Compute the sum  $\sum_{n=0}^{10} 2^n$ .

- A 1023.
- B 2047.
- C 2048.
- D 4095.
- E None of the above.

(2) What is the correct asymptotic behavior (order analysis) for the sum  $S(n) = \sum_{i=1}^n i$ .

- A  $S(n) \in O(n)$ .
- B  $S(n) \in o(n^2)$ .
- C  $S(n) \in \Theta(n^2)$ .
- D  $S(n) \in \omega(n^2)$ .
- E None of the above.

(3) Which is the correct asymptotic order relationship that describes the sum  $S(n) = \sum_{i=0}^{2n} 2^i$

- A  $S(n) \in \Theta(n^2)$ .
- B  $S(n) \in \Theta(2^n)$ .
- C  $S(n) \in \omega(2^n)$ .
- D  $S(n) \in o(2^n)$ .
- E None of the above.

(4) What is the greatest common divisor of 5292 and 6006,  $\gcd(5292, 6006)$ ?

- A 6.
- B 14.
- C 21.
- D 48.
- E None of the above.

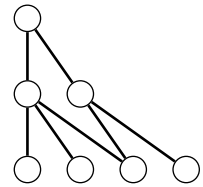
(5) What is the last digit of  $n$  (the remainder when divided by 10), where  $n = 29^9 - 21^7$ ?

- A 0
- B 2
- C 4
- D 6
- E 8

- (6) A friendship network (simple graph) has vertices having degree sequence  $\delta = [4, 4, 2, 2, 2]$ . How many edges (friendship links) are in this friendship network?
- A 6 edges
- B 7 edges
- C 8 edges
- D Not enough information to determine the number of edges
- E This friendship network cannot possibly exist

- (7) You wish to color the graph on the right so that linked vertices do not get the same color. What is the minimum number of colors needed (the chromatic number).

- A 2
- B 3
- C 4
- D 5
- E 6



- (8) Evaluate  $\binom{18}{7} - \binom{18}{11}$

- A 0.
- B 1.
- C 2.
- D 4.
- E 36.

- (9) How many subsets of  $\{a, b, c, d, e, f, g\}$  contain ***at most*** 3 elements.?

- A 32.
- B 64.
- C 128.
- D 256.
- E None of the above.

- (10) *Estimate* the number of possible social networks which could represent the friendship relationships among the 10 friends  $A, B, C, D, E, F, G, H, I, J$ . (How many different 10-vertex graphs are there?)

- A  $3.5 \times 10^{12}$ .
- B  $3.5 \times 10^{13}$ .
- C  $3.5 \times 10^{14}$ .
- D  $3.5 \times 10^{15}$ .
- E  $3.5 \times 10^{16}$ .

(11) In how many ways can you misspell TRIANGLE, assuming you use all the same letters?

- A  $8^8$ .
- B  $8^8 - 1$ .
- C  $8!$ .
- D  $8! - 1$ .
- E None of the above.

(12) In how many ways can you misspell SUCCESS, assuming you use all the same letters?

- A 379.
- B 399.
- C 419.
- D 449.
- E None of the above.

(13) In how many ways can 10 identical rings be placed on your 5 left-fingers (multiple rings can go on a finger).

- A  $5^{10}$ .
- B  $10^5$ .
- C  $\binom{14}{4}$ .
- D  $\binom{15}{5}$ .
- E None of the above.

(14) What is the coefficient of  $x^3$  in the expansion of  $(2\sqrt{x} + x)^5$ ?

- A 0.
- B 5.
- C 40.
- D 80.
- E 120

(15) How many of the numbers in  $\{1, 2, \dots, 1000\}$  are divisible by 10 or 11?

- A 177.
- B 179.
- C 181.
- D 183.
- E None of the above.

**2** Prove that  $n^2$  is divisible by 8 if and only if  $n$  is divisible by 4. ( $n \in \mathbb{N}$ )

**3** Prove or disprove. There exists  $x, y \in \mathbb{Z}$  for which  $2x^2 + 5y^2 = 14$ .

**4** (Diagonal Binomial Sum) Prove by *induction*:  $\sum_{k=0}^n \binom{10+k}{k} = \binom{10+n+1}{n}$  for all integers  $n \geq 0$ .

5 How many 10-bit binary strings contain 00 as a substring.



**6** Prove or disprove: Any graph has an even number of vertices having odd degree.  
(The graph is undirected and simple, i.e, has no loops or parallel edges.)

SCRATCH

SCRATCH