# MIDTERM: 90 Minutes

Last Name: _____

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
**NO COLLABORATION or electronic devices. Any violations result in an F.**
**NO questions** allowed during the test. Interpret and do the best you can.
You **MUST** show **CORRECT** work, **even on multiple choice questions**, to get credit.

## GOOD LUCK!

| 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|
| | | | | | | |
| **150** | **20** | **20** | **20** | **20** | **20** | **250** |

# INSTRUCTIONS

1. This is a **closed book** test. No electronics, books, notes, internet, etc.
2. **One** $8.5 \times 11$ **double sided crib sheet** is allowed.
3. The test will become available in Submitty at 8am on the test date.
4. Your PDF is due in Submitty by 2pm.
5. By submitting the test you attest that:
   – the work is entirely your own.
   – you obeyed the time limits of the exam.
6. Your submission *must* be typed and submitted as a PDF file.
7. The first page should list your 15 multiple choice answers, like:

| | |
|---|---|
| (1) | A |
| (2) | B |
| (3) | C |
| (4) | D |
| ⋮ | |
| (15) | A |

8. Start each of problems 2–6 on a new page. **SHOW WORK**.
9. After the answers, start a new page to show work for the multiple choice:

| | |
|---|---|
| (1) | Because $x$ is even |
| (2) | Because $\sqrt{2}$ is irrational. |
| (3) | Number of links is $1 + 2 + \cdots + 10 = 55$ |
| ⋮ | |
| (15) | Because we proved in class that $\ell = n - 1$ |

   – Some problems may be "easy", so give a one line justification.
   – Some problems may require a detailed reasoning.
   – correct answers: 10 points
   – wrong answers or no work/explanation: 0.
10. **If you don't show correct work, you won't get credit.**
11. Submit with plenty of time to spare. A late test won't be accepted.
    – We won't accept submissions that are even 1 second late.

## 1 Circle one answer per question. 10 points for each correct answer.

(1) What is the correct asymptotic behavior (order analysis) for the sum $S(n) = \sum_{i=1}^{n} \sqrt{i}$.

A $S(n) \in \Theta(n)$.

B $S(n) \in \Theta(n^2)$.

C $S(n) \in \Theta(n^3)$.

D $S(n) \in \Theta(n^4)$.

E None of the above.

(2) What is the correct asymptotic behavior (order analysis) for the sum $S(n) = \sum_{i=1}^{n^2} i$.

A $S(n) \in \Theta(n)$.

B $S(n) \in \Theta(n^2)$.

C $S(n) \in \Theta(n^3)$.

D $S(n) \in \Theta(n^4)$.

E None of the above.

(3) Which is the correct asymptotic order relationship that describes the sum $S(n) = \sum_{i=0}^{n} 2^i$

A $S(n) \in \Theta(n^2)$.

B $S(n) \in \Theta(2^n)$.

C $S(n) \in \omega(2^n)$.

D $S(n) \in o(2^n)$.

E None of the above.

(4) Estimate $\ln(10^9!)$, that is the logarithm of the factorial of $10^9$.

A $2 \times 10^{10}$.

B $2 \times 10^{20}$.

C $2 \times 10^{30}$.

D $2 \times 10^{40}$.

E $2 \times 10^{50}$.

(5) $\gcd(210, 385) = 210x + 385y$ where $x, y \in \mathbb{Z}$. What are a possible choice for $x, y$?

A $x = 1, \quad y = 1$.

B $x = -1, \quad y = 1$.

C $x = 2, \quad y = -1$.

D $x = 1, \quad y = 1$.

E None of the above.

(6) What is the remainder when $29^{2019} - 22^{2019}$ is divided by 3?

A 0

B 1

C 2

D 3

E None of the above.

(7) A friendship network (simple graph) has vertices having degree sequence $\boldsymbol{\delta} = [5, 4, 3, 2, 2]$. How many edges (friendship links) are in this friendship network?

A 6 edges

B 7 edges

C 8 edges

D Not enough information to determine the number of edges

E This friendship network cannot possibly exist

(8) You wish to color the graph on the right so that linked vertices do not get the same color. What is the minimum number of colors needed (the chromatic number).

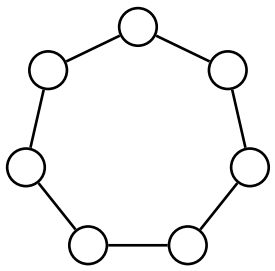
A 2

B 3

C 4

D 5

E 6



(9) At a party with $n$ people, everyone shakes hands with everyone else. How many handshakes occur?

A $\frac{1}{2}n(n-1)$.

B $\frac{1}{2}n(n+1)$.

C $2n$.

D $n^2$.

E None of the above.

(10) A connected graph has $n \geq 2$ vertices and no cycles. Does the graph have a degree 1 vertex?

A Yes, always.

B No, never.

C If $n$ is even yes, otherwise no.

D If $n$ is even no, otherwise yes.

E None of the above.

(11) How many subsets of $\{1, 2, 3, 4, 5, 6, 7\}$ contain ***one*** even number.?

A $3 + 2^4$.

B $3 \times 2^4$.

C $\binom{7}{3}$.

D $2^7$.

E None of the above.

(12) There are 4 candy colors. How many goody-bags with 10 candies can you make.

A $10^4$.

B $4^{10}$.

C $\binom{10}{4}$.

D $\binom{14}{4}$.

E None of the above.

(13) How many *sequences* of four non-negative integers add up to 10? That is non-negative integers $x_1, x_2, x_3, x_4$ with $x_1 + x_2 + x_3 + x_4 = 10$, for example $(4, 3, 2, 1), (3, 4, 2, 1)$.

A $10^4$.

B $4^{10}$.

C $\binom{10}{4}$.

D $\binom{14}{4}$.

E None of the above.

(14) In how many ways can you misspell TEDDY, assuming you use all the same letters?

A 57.

B 58.

C 59.

D 60.

E None of the above.

(15) How many of the 1,000 numbers in $\{0, 1, \ldots, 999\}$ contain the digits 1 or 2?

A 460.

B 474.

C 488.

D 512.

E None of the above.

**2** **Prove that $\log_2 9$ is not a rational number.**

**3** **Prove by induction for all $n \geq 1$: $\sum_{i=1}^{n} i2^i = (n-1)2^{n+1} + 2$.**

**4** **Find a formula for $A_n$ and prove your answer. $A_0 = 1$ and $A_n = A_{n-1} + n$ for $n \geq 1$**

**5 How many subsets of $\{1, 2, 3, \ldots, 10\}$ have an even sum. (The empty set $\phi$ has even sum.)**
For example, the subset $\{2, 6\}$ has even sum but $\{2, 7\}$ has odd sum.

## 6 FOCSbook has $n \geq 2$ people and each person has $n-2$ friends. Find all possible $n$.

Precisely state your result and prove it. TINKER!

For the $n$ where it's possible, you must show how. For the $n$ where it's impossible, you must prove it.

## SCRATCH

## SCRATCH