MIDTERM: 90 Minutes

Last Name: _____

First Name: _____

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Section: _____

Answer **ALL** questions. Open Orange DMC-textbook. Nothing else.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.
You **MUST** show **CORRECT** work, **even on multiple choice questions**, to get credit.

GOOD LUCK!

1	2	3	4	5	Total
150	**25**	**25**	**25**	**25**	**250**

1 Circle one answer per question. 10 points for each correct answer.

(1) Give the negation of "Attending class is necessary to pass the course".

A IF you did not pass the course THEN you did not attend class.

B IF you did not attend class THEN you did not pass the course.

C You did attend class AND you did not pass the course.

D You did not attend class AND you did pass the course.

E None of the above.

(2) Compute $S = \sum_{i=1}^{4} \sum_{j=1}^{4} (ij)$.

A $S = 50$.

B $S = 100$.

C $S = 200$.

D $S = 400$.

E None of the above.

(3) Estimate this sum $S = \sum_{i=0}^{5} \sum_{j=0}^{10} 2^{i+j}$.

A $S = 3.2 \times 10^3$.

B $S = 1.3 \times 10^4$.

C $S = 2.6 \times 10^4$.

D $S = 1.3 \times 10^5$.

E $S = 2.6 \times 10^5$.

(4) Let $S(n) = 1 + 2 + 3 + \cdots + n$. Which is true?

A $S(n) \in \Theta(n)$.

B $S(n) \in \Theta(n \log_2 n)$.

C $S(n) \in \Theta(n^2)$.

D $S(n) \in \Theta(2^n)$.

E None of the above.

(5) Compute $\gcd(210, 385)$. That is, compute the greatest common divisor of 210 and 385.

A 3.

B 5.

C 7.

D 9.

E None of the above.

(6) Compute the remainder when 6^{2025} is divided by 7.

A 1.

B 3.

C 5.

D 6.

E None of the above.

(7) A friendship network (simple graph) has degree sequence $[4, 4, 3, 3, 2]$. How many edges does it have?

A 7.

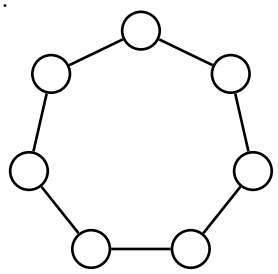
B 8.

C 9.

D 10.

E None of the above, or not enough information.

(8) You wish to color the graph on the right so that linked vertices do not get the same color. What is the minimum number of colors needed (the chromatic number).



A 2

B 3

C 4

D 5

E 6

(9) What is the minimum number of children that guarantees at least two have the first and last initial 'A'?

A 26

B $26 + 1$

C 26^2

D $26^2 + 1$

E None of the above.

(10) A race has 10 runners. In how many ways can the 10 runners finish the race?

A 10^{10}

B $\binom{10}{5}$

C $10 \times 9 \times 8$

D $10!$

E None of the above.

(11) You flip a coin. You then flip a second coin. You roll a die. You then roll a second die. How may possible outcomes of this experiment are there?

A 4.

B 12.

C 36.

D 144.

E None of the above.

(12) You have 10 different books. In how many ways can you give Alice 3 books and then give Bob 2 books?

A $\binom{10}{5}$.

B $10!/(2! \times 3! \times 5!)$.

C $10!/5!$.

D 10^5.

E None of the above.

(13) How many 5 bit strings begin with 1 or have at least 3 ones?

A 5.

B 16.

C 21.

D 32.

E None of the above.

(14) How many subsets of the set $\{1, 2, 3, 4, 5\}$ have size 2 or 3?

A 5.

B 10.

C 20.

D 32.

E None of the above.

(15) How many 10-bit strings contain 00 as a substring [Hint: count strings *not* containing 00 as a substring].

A $\binom{10}{8}$.

B 2^8.

C 880.

D 1024.

E None of the above.

2 **Prove. In a right triangle with integer sides, the two shorter sides cannot both be odd.**

3 **Prove *by induction.*** $\sum_{i=1}^{n}(i+1)2^i = n \times 2^{n+1}$**, for** $n \geq 1$**.**

4 **Compute a formula for this sum,** $S(n) = \sum_{i=0}^{n} \sum_{j=0}^{n} (2^i + 2^j)^2$**.**

5 **Prove the binomial theorem *by induction* [Hint: Pascal's Identity]. For $x, y \in \mathbb{R}$ and $\forall n \geq 1$**

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}.$$

SCRATCH

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