QUIZ 1: 90 Minutes

Last Name: ______________________

First Name: ______________________

RIN: ______________________

Section: ______________________

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

Total
100

1. Which of the following would show that $p \rightarrow q$ is true?

$\boxed{\text{A}}$ Assume p is not true and show q is not true.

$\boxed{\text{B}}$ Show p is always true.

$\boxed{\text{C}}$ Show p is always false.

$\boxed{\text{D}}$ Assume q is true and show p is not true.

2. $p \rightarrow (q \wedge r)$ is equivalent to what other compound proposition:

$\boxed{\text{A}}$ $(p \rightarrow q) \wedge r$

$\boxed{\text{B}}$ $(p \rightarrow q) \wedge (p \rightarrow r)$

$\boxed{\text{C}}$ $(p \wedge q) \rightarrow r$

$\boxed{\text{D}}$ $p \vee (q \wedge r)$

3. Which reasoning is correct in the deductions below?

$\boxed{\text{A}}$ If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

$\boxed{\text{B}}$ Everyone who eats apples is healthy. Malik is not healthy. Therefore, Malik does not eat apples.

$\boxed{\text{C}}$ At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

$\boxed{\text{D}}$ Lights are turned on in the evenings. It is daytime. Therefore, the lights are turned off.

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \rightarrow (P(n^2) \wedge P(n-2))$ is true for $n \geq 2$. For which n can we be <u>**sure**</u> $P(n)$ is true?

$\boxed{\text{A}}$ All $n \geq 2$.

$\boxed{\text{B}}$ All even $n \geq 0$.

$\boxed{\text{C}}$ All odd $n \geq 0$.

$\boxed{\text{D}}$ All n which are perfect squares.

5. You may take as known facts: $0 = 0$ and the standard operations of algebra from high-school math. Which of the following is a valid proof that $7 = 7$.

(I)

$$\begin{array}{ll} 1. & 7=7 \\ 2. & 7-7=7-7 \\ 3. & 0=0 \quad \checkmark \\ \hline \rightarrow & 7=7 \end{array}$$

(II)

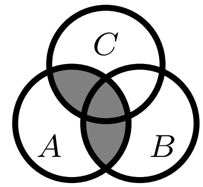
$$\begin{array}{ll} 1. & 7\neq 7 \\ 2. & 7-7\neq 7-7 \\ 3. & 0\neq 0 \quad \textbf{!FISHY} \\ \hline \rightarrow & 7=7 \end{array}$$

(III)

$$\begin{array}{ll} 1. & 0=0 \\ 2. & 0+7=0+7 \\ 3. & 7=7 \quad \checkmark \\ \hline \rightarrow & 7=7 \end{array}$$

$\boxed{\text{A}}$ I & II & III. $\boxed{\text{B}}$ I & II $\boxed{\text{C}}$ II & III. $\boxed{\text{D}}$ I & III.

6. Which expression represents the shaded region in the Venn diagram:



$\boxed{\text{A}}$ $A \cap B \cap C$ $\boxed{\text{B}}$ $A \cap (B \cup C)$ $\boxed{\text{C}}$ $A \cup (B \cap C)$ $\boxed{\text{D}}$ $A \cup B \cup C$

7. The domain of x, y is $\mathbb{R}$. True or false, $\exists x : (\forall y : xy = y)$?

$\boxed{\text{A}}$ True.

$\boxed{\text{B}}$ False.

$\boxed{\text{C}}$ Can't say because it depends on x.

$\boxed{\text{D}}$ Can't say because it depends on y.

8. T_n satisfies a recurrence $T_0 = 3$; $T_n = 2T_{n-1}$ for $n \geq 1$. Give a formula for T_n.

$\boxed{\text{A}}$ $T_n = 3(n+1) + \frac{3}{2}n(n-1)$

$\boxed{\text{B}}$ $T_n = 3 \cdot 2^{n+1} - 3(n+1)$

$\boxed{\text{C}}$ $T_n = 3 \cdot 2^n$

$\boxed{\text{D}}$ $T_n = 2^n$

9. The set $\mathcal{A}$ of arithmetic strings using characters in the set $\Sigma = \{1, +, \times, (,)\}$ has a recursive definition:

$$\begin{array}{rl} \textbf{[Base Case:]} & 1 \in \mathcal{A}; \\ \textbf{[Constructor Rules:]} & x, y, z \in \mathcal{A} \ \rightarrow (x + y + z) \in \mathcal{A} \\ & \quad x, y \in \mathcal{A} \ \rightarrow (x \times y) \in \mathcal{A}. \end{array}$$

Which string is in $\mathcal{A}$

$\boxed{\text{A}}$ $(1+1+1) \times (1+1)$

$\boxed{\text{B}}$ $(1+1+1) \times ((1+1+1)+1+1)$

$\boxed{\text{C}}$ $((1+1+1) \times ((1+1+1)+1+1))$

$\boxed{\text{D}}$ $((1 \times 1)+1+1+1)$

10. There are 5 rooted binary trees with 3 nodes. How many are there with 4 nodes?

$\boxed{\text{A}}$ 7

$\boxed{\text{B}}$ 12

$\boxed{\text{C}}$ 14

$\boxed{\text{D}}$ 16

SCRATCH