QUIZ 2: 90 Minutes

Last Name: _____

First Name: _____

RIN: _____

Section: _____

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

Total
100

1. Assume a year has 360 days and is composed of 12 months each of 30 days. How many people do you need in a room to **guarantee** that at least two people share the same *birthday month*?

A 12 B 13 C 14 D $11 \times 30 + 1 = 331$ E 360

2. Shirts come in 4 colors. You need to assign shirts to 5 students. In how many ways can you do this?

A 0 B 4^5 C 5^4 D 4! E 5!

3. Shirts come in 4 colors. You need to assign shirts to 5 students, and no two students can get the same color shirt. In how many ways can you do this?

A 0 B 4^5 C 5^4 D 4! E 5!

4. Shirts come in 4 colors. 5 students are in a row. You need to assign shirts to the students, and two students standing next to each other cannot get the same color shirt. In how many ways can you do this?

A 0 B $9 \times 8 \times 7 \times 6 \times 5$ C $\binom{9}{5}$ D $\binom{5}{4}$ E 4×3^4

5. x_1, x_2, x_3, x_4 are *natural numbers* $(1, 2, \ldots)$. In how many different ways can you choose x_1, x_2, x_3, x_4 so that $x_1 + x_2 + x_3 + x_4 = 10$? For example, two *different* solutions are (1,2,3,4) and (2,1,3,4).

A 54 B 64 C 74 D 84 E 98

6. You roll a pair of fair dice. What is the probability that the sum is even?

A $\frac{1}{6}$ B $\frac{12}{36}$ C $\frac{16}{36}$ D $\frac{2}{5}$ E $\frac{1}{2}$

7. You roll a pair of fair dice. What is the probability that the sum is even *given* that the two values rolled are different?

A $\frac{1}{6}$ B $\frac{12}{36}$ C $\frac{16}{36}$ D $\frac{2}{5}$ E $\frac{1}{2}$

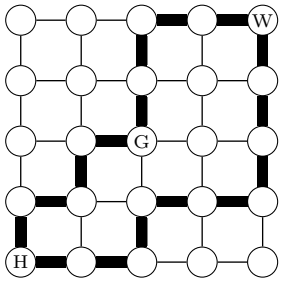
8. You independently generate the 4 bits of a binary sequence $b_1b_2b_3b_4$ with $\mathbb{P}[b_i = 0] = \frac{1}{2}$. Compute the probability that $\sum_{i=1}^{4} b_i = 2$

A $\frac{1}{16}$ B $\frac{2}{16}$ C $\frac{4}{16}$ D $\frac{6}{16}$ E $\frac{8}{16}$

9. Problems 9 and 10 refer to the grid graph on the right.

There are three special nodes in this graph: (H) is home; (W) is work; (G) is the grocery store. Two shortest paths from Work to Home are highlighted in the graph, one goes through the grocery store and one does not. All shortest paths from (W) to (H) have length 8.

How many *different* shortest paths from (W) to (H) are there (two paths are different if there is an edge in one path that is not used in the other)?



A 40
B 2^8
C 50
D 60
E 70

10. If you randomly choose one of the shortest paths from (W) to (H), with each shortest path being equally likely, what is the probability that you will be able to pick up groceries on your way home from work.

A $\frac{18}{35}$
B $\frac{1}{2}$
C $\frac{11}{30}$
D $\frac{12}{25}$
E $\frac{13}{20}$

SCRATCH