QUIZ 1: 60 Minutes

Last Name: Solutions

First Name: ____________

RIN: ____________

Section: ____________

Answer ALL questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer

Total
100

1. C
2. A
3. D
4. C
5. C
6. C
7. D
8. C
9. D
10. D

1. We know that p is false. We do not know the truth value of q. Which of the following *must* be true?

   (I) $\neg p \vee \neg q$ (II) $\neg p \wedge \neg q$ (III) $\neg(p \wedge q)$ (IV) $p \rightarrow q$

   T ? F T F T

   [A] I, II, III

   [B] I, II, IV

   [C] I, III, IV

   [D] II, III, IV

C

2. For a set of horses $\mathcal{H}$, determine whether the following claim is true or false:
   IF every subset of 10 horses has the same color, THEN every subset of 11 horses has the same color.

   [A] Always true no matter what $\mathcal{H}$ is.

   [B] Always false no matter what $\mathcal{H}$ is.

   [C] Not enough information to determine whether it is true or false.

   [D] False if $\mathcal{H}$ has fewer than 11 horses but true otherwise.

A

Any 11 must have the same color because any subset of 10 do.
If there aren't 11 horses then it is trivially true.

3. Which reasoning is correct in the deductions below?

   [A] If it rains, then Kilam brings an umbrella. It did not rain. Therefore, Kilam did not bring an umbrella.

   [B] Everyone who eats apples is healthy. Malik is healthy. Therefore, Malik eats apples.

   [C] At the party you can have cake or ice-cream. You had cake. Therefore, you did not have ice-cream.

   [D] Lights are turned on in the night. Lights are off. Therefore, it is day.

   $p \rightarrow q$ $\sim q$ $\therefore \sim p$

D

4. $P(n)$ is a predicate (n is an integer). $P(2)$ is true; and, $P(n) \rightarrow P(n+2)$ is true for $n \geq 0$. For which n can we be sure $P(n)$ is true?

   [A] All $n \geq 2$.

   [B] All even $n \geq 0$.

   [C] All even $n \geq 2$.

   [D] All n which are perfect squares.

   $P(0) \rightarrow \boxed{P(2)} \rightarrow P(4) \rightarrow P(6) \rightarrow \ldots$

C

5. Which of the following, if any, is a valid way to prove $P(n) \rightarrow P(n+1)$.

   ~~Direct~~ Contraposition

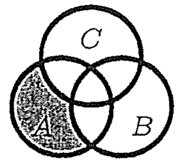
   (I) Let's see what happens if $P(n+1)$ is T.
   $\vdots$ (valid derivations)
   Look! $P(n)$ is T. ✓

   (II) Let's see what happens if $P(n+1)$ is F.
   $\vdots$ (valid derivations)
   Look! $P(n)$ is F. ✓

   [A] None [B] I [C] II [D] I and II

C

6. Which expression represents the shaded region in the Venn diagram:



$A \cap \overline{(B \cup C)}$
$= A \cap \overline{B} \cap \overline{C}$

C

$\boxed{A}$ $A \cap B \cap C$ $\boxed{B}$ $A \cap (B \cup C)$ $\boxed{C}$ $A \cap \overline{B} \cap \overline{C}$ $\boxed{D}$ $\overline{A} \cap B \cap C$

7. What is the more formal way to say: "There's a soul-mate for everyone"?

$\boxed{A}$ $\exists x \in$ PEOPLE : ($\exists y \in$ PEOPLE : x is a soul-mate for y) There is a pair of soul-mates

$\boxed{B}$ $\exists x \in$ PEOPLE : ($\exists y \in$ PEOPLE : y is a soul-mate for x) There is a pair of soulmates

D.

$\boxed{C}$ $\forall x \in$ PEOPLE : ($\forall y \in$ PEOPLE : y is a soul-mate for x) ~~There~~ All pairs of people are soulmates

$\boxed{D}$ $\forall x \in$ PEOPLE : ($\exists y \in$ PEOPLE : y is a soul-mate for x) ~~There~~ All people (x) have a soulmate (y)

8. T_n satisfies a recurrence $T_0 = 2$; $T_n = T_{n-1} + 3n$ for $n \geq 1$. Compute T_{100}.

$\boxed{A}$ 10,002

$\boxed{B}$ 10,102

C

$\boxed{C}$ 15,152

$\boxed{D}$ 14,002

$T_n = \cancel{T_{n-1}} + 3n$

$\cancel{T_{n-1}} = \cancel{T_{n-2}} + 3(n-1)$ $= T_n = T_0 + 3(1+2+\dots n)$ $n = 100.$

$= 2 + 3 \cdot \frac{n(n+1)}{2}$

$\cancel{T_1} = T_0 + 3$

$= 2 + 3 \cdot \underbrace{50 \cdot 101}_{5050} = 2 + 15150 = 15152$

9. Determine the set $\mathcal{A}$ defined recursively by:

① $1 \in \mathcal{A}$. [basis]

② $x, y \in \mathcal{A} \rightarrow x + y \in \mathcal{A}$ [constructors]
$x, y \in \mathcal{A} \rightarrow x - y \in \mathcal{A}$. $0 \in A$ $-1 \in A$

③ Nothing else is in $\mathcal{A}$. [minimality]

D

~~$\boxed{A}$~~ $\mathcal{A} = \{1, 2, 3, \dots\}$

$\boxed{B}$ $\mathcal{A} = \{0, 1, 2, 3, \dots\}$

~~$\boxed{C}$~~ $\mathcal{A} = \{\pm 1, \pm 2, \pm 3, \dots\}$

$\boxed{D}$ $\mathcal{A} = \{0, \pm 1, \pm 2, \pm 3, \dots\}$

10. ① $1 \in \mathcal{S}$. [basis] This is a recursive definition of a set $\mathcal{S}$ *without*
② $x \in \mathcal{S} \rightarrow x + 1 \in \mathcal{S}$. [constructor] the minimality clause "Nothing else is in $\mathcal{S}$."

Which of the following *cannot* be the set $\mathcal{S}$

$\boxed{A}$ $\mathbb{N}$ ✓ $\{1, 2, 3 \dots\}$ ✓ satisfies ①, ②

$\boxed{B}$ $\mathbb{Z}$ ✓ $\{0, \pm 1, \pm 2, \dots\}$ ✓ satisfies ①, ②

D.

$\boxed{C}$ $\mathbb{N} \cup \{x \mid x = n + \frac{1}{2}, n \in \mathbb{N}\}$ ✓ $\{1, 2, 3, \dots\} \cup \{1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, \dots\}$ satisfies ①, ② ✓

$\boxed{D}$ $\mathbb{N} \cup \{\frac{1}{2}\}$ $\frac{1}{2} \in S \rightarrow \frac{3}{2} \in S$ ② ✗