

# QUIZ 1: 60 Minutes

Last Name: \_\_\_\_\_

First Name: \_\_\_\_\_

RIN: \_\_\_\_\_

Section: \_\_\_\_\_

Answer **ALL** questions.

**NO COLLABORATION** or electronic devices. Any violations result in an **F**.

**NO questions** allowed during the test. Interpret and do the best you can.

## GOOD LUCK!

Circle at most one answer per question.

**10 points** for each correct answer.

You **MUST** show work to ensure getting full credit.

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| <b>Total</b> |
|              |
| <b>150</b>   |

1.  $\sqrt{2}$  is what kind of number?
- A A natural number.
  - B An integer.
  - C A rational number.
  - D A member of the set  $\mathbb{Q}$ .
  - E None of the above.
2. What is the set  $\mathbb{Z} \cap \overline{\mathbb{N}} \cap \mathcal{S}$ , where  $\mathcal{S}$  is the set of perfect square numbers. The universal set is  $\mathbb{R}$ .
- A  $\emptyset$ , the empty set.
  - B  $\{0\}$ .
  - C  $\mathcal{S}$ .
  - D The non-positive integers.
  - E The set is not well defined.
3.  $A = \{2, 5\}$  and  $B = \{3, 7\}$ . What is the Cartesian Product  $A \times B$ ?
- A  $\{6, 14, 15, 35\}$ .
  - B  $\{2, 3, 5, 7\}$ .
  - C  $\{(2, 3), (2, 7), (5, 3), (5, 7)\}$ .
  - D  $\{(2, 3), (3, 2), (2, 7), (7, 2)(5, 3), (3, 5), (5, 7), (7, 5)\}$ .
  - E None of the above.
4. How many rows in the truth table of  $(p \rightarrow q) \vee p$  are T?
- A 0.
  - B 1.
  - C 2.
  - D 3.
  - E 4.
5. IF (you ace the final OR the quiz), THEN you get an A. You did get an A. *Did you ace the final?*
- A Yes, for sure.
  - B No, for sure.
  - C Yes, if and only if you did not ace the quiz.
  - D Yes if you did not ace the quiz; otherwise we don't know.
  - E None of the above.

6. Which mathematical claims are T. Note,  $(a, b, c) \in \mathbb{R}^3$  stands for triples of real numbers  $(a, b, c)$ .

(I) IF  $(\forall(a, b, c) \in \mathbb{R}^3 : ax^2 + bx + c = 0)$ , THEN  $x = 0$

(II)  $\forall(a, b, c) \in \mathbb{R}^3 : (\text{IF } ax^2 + bx + c = 0, \text{ THEN } x=0)$

A I only.

B II only.

C Both I and II.

D Neither I nor II.

7. For a natural number  $n$ , consider the implication: IF  $n \geq n + 1$ , THEN  $n+1 \geq n + 2$   
Determine whether the *implication* is T or F?

A Always T no matter what  $n$  is.

B Always F no matter what  $n$  is.

C T only for positive  $n$ .

D T only for negative  $n$ .

E None of the above.

8. What method of proof is used to prove that  $\sqrt{2}$  is irrational?

A Direct proof.

B Contraposition proof.

C Proof by contradiction.

D Induction.

E Strong induction.

9. Which gives a valid proof of the implication  $(p \vee q) \rightarrow r$ .

A Assume  $p$  is T and show that  $r$  must be T.

B Assume  $q$  is T and show that  $r$  must be T.

C Assume  $r$  is F and show that  $p$  must be F.

D Assume  $r$  is F and show that  $q$  must be F.

E None of the above.

10.  $P(n) =$  “ $n$  is even” and  $Q(n) =$  “ $n$  is a sum of two primes”. Translate “ $\forall n \in \mathbb{N} : P(n) \rightarrow Q(n)$ .”

A If  $n$  is a natural number then  $n$  is a sum of two primes.

B Every prime number is a natural number.

C There is a natural number which is a prime number.

D Every positive even number is a sum of two primes.

E Some positive even number is a sum of two primes.

11.  $P(n)$  is a predicate ( $n$  is an integer).  $P(1)$  is true; and,  $P(n) \rightarrow P(2n - 1) \wedge P(2n)$  is true for  $n \geq 1$ . Which set captures *all*  $n$  for which we can be sure  $P(n)$  is T?
- A All  $n \geq 1$ .
- B All  $n \geq 2$ .
- C All even  $n \geq 1$ .
- D All even  $n \geq 2$ .
- E None of the above.
12. Which of the following, if any, is a valid way to prove  $P(n) \rightarrow P(n + 1)$  in an induction proof.
- |  |   |   |   |
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| (I) Let's see what happens if $P(n)$ is T.<br>$\vdots$ (valid derivations)<br>Look! $P(n + 1)$ is T. | ✓ | (II) Let's see what happens if $P(n + 1)$ is F.<br>$\vdots$ (valid derivations)<br>Look! $P(n)$ is F. | ✓ |
|--|---|---|---|
- A None.                       B I only.                       C II only.                       D Both I and II
13. We wish to break a group of  $n$  students into project-teams of 4 or 7 students.
- A IF  $n \geq 7$ , THEN it can be done.
- B IF  $n \geq 11$ , THEN it can be done.
- C IF  $n \geq 14$ , THEN it can be done.
- D IF  $n \geq 19$ , THEN it can be done.
- E None of the above are T.
14.  $A = \{x \mid x = 12m + 21n, \text{ for } m, n \in \mathbb{Z}\}$ . T or F:  $A = \mathbb{Z}$ ?
- A T.
- B F.
- C Depends on  $m$ .
- D Depends on  $n$ .
- E None of the above.
15. What is the function defined recursively on the right for integer  $n \geq 0$ .
- |   |   |
|---|---|
| <input type="checkbox"/> A $f(n) = n!$ .<br><input type="checkbox"/> B $f(n) = 2^n$ .<br><input type="checkbox"/> C $f(n) = 2^n \times n^n$ .<br><input type="checkbox"/> D $f(n) = 2^n \times n!$ .<br><input type="checkbox"/> E None of the above. | $f(n) = \begin{cases} 1 & n = 0; \\ 2nf(n - 1) & n \geq 1. \end{cases}$ |
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SCRATCH