QUIZ 1: 60 Minutes

Last Name: Solutions
First Name: ______
RIN: ______
Section: ______

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.

When in doubt, TINKER.

Total
200

1. The set $S = \{(-1)^k \mid k \in \mathbb{N}\}$ could be which of the sets below?

$S = \{-1, 1, -1, 1, -1, 1, -1, 1, \ldots\}$

A $\{1, 2, 3, 4, 5, 6, 7, 8, \ldots\}$.

B $\{-1, 2, -3, 4, -5, 6, -7, 8, \ldots\}$.

C $\{1, 1\}$.

D $\{1, -1, \}$.

E None of the above.

repetition is removed in a set.

D

2. The complement of a set S is denoted $\overline{S}$. Which set is the same as $\overline{A \cap B}$?

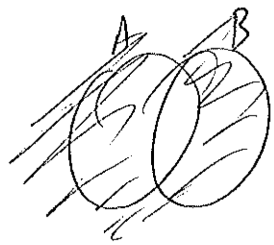
A $A \cap B$.

B $A \cup B$.

C $\overline{A} \cap \overline{B}$.

D $\overline{A} \cup \overline{B}$.

E None of the above.



$= \overline{A} \cup \overline{B}$

D

3. Rewrite the sentence "Attending class is necessary to pass the course" using IF ... THEN ... form.

A IF you did attend class THEN you did pass the course.

B IF you did not attend class THEN you did not pass the course.

C IF you did not pass the course THEN you did not attend class.

D IF you did not attend class THEN you did pass the course.

E None of the above.

pass course → attend class.
same as:
don't attend class → do not pass

B

4. Give the negation of "IF you did not eat your peas THEN you did not get candy".

~~A~~ IF you did eat your peas THEN you did get candy.

~~B~~ IF you did get candy THEN you did eat your peas.

~~C~~ IF you did not get candy THEN you did not eat your peas.

~~D~~ You did eat your peas AND you did not get candy.

E None of the above.

$\neg(p \to q)$ is true
if and only if
$p \to q$ is F
so p is T and q is F

Same as $p \wedge \neg q$
"did not eat peas and did get candy"

E

5. How many rows are in the truth table of $(p \vee \neg q) \to (p \wedge q)$ are true?

A 1.

B 2.

C 3.

D 4.

E None of the above.

p	q	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
F	F	T	F	F
F	T	F	F	T
T	F	T	F	F
T	T	T	T	T

B

Work!

6. IF you sing THEN you don't swim. Joe doesn't sing; Jim swims. What else do we know?

A We know Joe swims.

B We know Joe doesn't swim.

C We know Jim sings.

D We know Jim doesn't sing.

E None of the above.

D

Joe: $p \to q$
$\neg p \to ?$
Jim: $p \to q$
$\neg q \to \neg p$
Jim doesn't sing

7. Let E be the set of even numbers greater than 2 and let P be the set of primes. Formulate mathematically the claim: "Every even number greater than two is a sum of two primes."

A $\forall e \in E : (\exists p, q \in P : e = p + q)$.

B $\exists e \in E : (\forall p, q \in P : e = p + q)$.

C $\forall p, q \in P : (\exists e \in E : e = p + q)$.

D $\exists p, q \in P : (\forall e \in E : e = p + q)$.

E None of the above.

A

$\forall e \in E : (\exists p,q \in P : e = p+q)$

8. Which proof-method is **not acceptable** to prove $p \to q$?

A Prove p is false. ✓

B Assume p is true and prove q is true. ✓ Direct

C Assume q is false and prove p is false. ✓ Contraposition

D Assume p is true and assume q is false. Now derive a contradiction. ✓ can't happen

E All of the above are valid ways to prove $p \to q$.

E

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

9. Consider the claim $\exists m, n \in \mathbb{N} : n^2 - 4m = 2$. Is the claim true or false?

A True.

B False.

C It depends on m.

D It depends on n.

E None of the above.

B

Work!

$n^2 - 4m = 2$
$\to n^2 = 2 + 4m$ (even $\to$ n is even $\to n = 2k$.
$4k^2 - 4m = 2$
LHS div by 4
RHS not div by 4
FISHY $\therefore$ not possible.

10. What is the minimum number of L-tiles need to cover the 3×3 patio. Tiles may overlap.

A 2

B 3

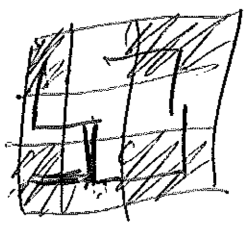
C 4

D 5

E None of the above.

C

Work!



No 2 shaded squares can be covered by the same tile
$\to$ 4 tiles needed.

A

11. How would you disprove: for every $n \in \mathbb{N}$, $3^n + 2$ is prime?

[A] Find a value $n_* \in \mathbb{N}$ for which $3^{n_*} + 2$ is not prime. ← Counter Example.

[B] Show that $3^n + 2$ is prime for $n = 1, 2, 3$.

[C] Proof by induction.

[D] Direct proof.

[E] None of the above.

C

12. You have 2¢ and 9¢ stamps for making postage. Which claim is true.

[A] You can make any postage greater than 1¢.

[B] You can make any postage greater than 5¢..

[C] You can make any postage greater than 15¢.

[D] There is no postage greater than 15¢ that you can make.

[E] None of the above.

$P(n) \to P(n+2)$

$9 = 9¢$ $\quad 10 = 5 \times 2¢$ ∴ Can Make ≥ 9

Can't make 3 or 6

so C.

E

13. How do you prove, by induction, the claim "The last digit of 6^n is 6" for all $n \geq 1$?

[A] Show the last digit of 6^2 is 6.

[B] Show the last digit of $6^2, 6^3, 6^4$ all the way up to $6^{1,000,000}$ is 6.

[C] Show, for $n \geq 1$, if the last digit of 6^n is 6 then the last digit of 6^{n+1} is 6.

[D] Show the last digit of 6^2 is 6 and show, for $n \geq 1$, if the last digit of 6^n is 6 then the last digit of 6^{n+1} is 6.

[E] None of the above. ← wrong base case

D

Work!

14. For $n \geq 1$, give a formula for $S(n) = 1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + (n-1) \times n + n \times (n+1)$.

[A] $S(n) = 2n(n+4)(2n+1)/15$.

[B] $S(n) = n(n+3)(3n+2)/10$.

[C] $S(n) = n(n+5)(5n+2)/21$.

[D] $S(n) = n(n+1)(n+2)/3$.

[E] None of the above.

→ only one that works for $S(1), S(2), S(3)$.

→ not integer for $S(3)$

Proof by induction

$S(1) = 2$ ✓

$S(n) = \frac{n(n+1)(n+2)}{3}$

$S(n+1) = S(n) + (n+1)(n+2)$

$= \frac{n(n+1)(n+2)}{3} + (n+1)(n+2)$

$= (n+1)(n+2)\left[\frac{n}{3} + 1\right]$

$= \frac{1}{3}(n+1)(n+2)(n+3)$ ✓ ∎

C

15. $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n-3) + 3$ for $n > 3$. What is $f(100)$?

[A] It cannot be computed because the recursion does not have enough base cases.

[B] 50.

[C] 100.

[D] 200.

[E] None of the above.

→ need 3 base cases } So function is not well defined.

Cannot compute $f(6)$

But can still compute $f(100)$ because base case $f(3)$ is not needed.

$f(1)$	$f(4)$	$f(7)$	$f(10)$	...	$f(100)$
1	4	7	10		100

$100 = 1 + 3 \times 33$ in 33 steps

$f(100) \to f(1)$ takes each time adding 3.

A: 5 points

Work!

16. Define $\mathcal{A}$ recursively: (i) $1 \in \mathcal{A}$ (ii) $x \in \mathcal{A} \to 3x \in \mathcal{A}$ (iii) Nothing else is in $\mathcal{A}$. Which is true?

[A] Every number in $\mathcal{A}$ is a multiple of 3. ✗ $1 \in A$

[B] Every multiple of 3 is in $\mathcal{A}$. ✗ $6 \notin A$

[C] Every number in $\mathcal{A}$ is divisible by 3^i for some $i \in \mathbb{Z}$. ✓

[D] Every number divisible by 3^i for some $i \in \mathbb{Z}$ is in $\mathcal{A}$. ✗ $\frac{1}{3} \notin A$

[E] None of the above.

$A = \{1, 3, 3^2, 3^3, \ldots\}$
$= \{3^0, 3^1, 3^2, \ldots\}$

C

17. An RBT has 100 vertices. How many links does it have?

[A] 50.

[B] 99.

[C] 100.

[D] 101.

[E] None of the above or not enough information to tell.

# vertices = # links + 1
100 = 99 + 1

B

18. Rewrite the sum $1 + 3 + 5 + \cdots + 97 + 99$ using summation notation.

C

$1+2+\cdots+100$ $3+5+\cdots+101$ $1+3+5+\cdots+99$ $-1+1+3+\cdots+97$

[A] $\sum_{i=1}^{100} i$. [B] $\sum_{i=1}^{50}(2i+1)$. [C] $\sum_{i=0}^{49}(2i+1)$. ✓ [D] $\sum_{i=0}^{49}(2i-1)$. ✗ [E] ~~None of~~ them.

19. Compute $2 + 4 + 6 + \cdots 1000$.

[A] 500,000.

[B] 500,500.

[C] 250,000.

[D] 250,500.

[E] None of the above.

$2(1+2+\cdots+500)$

$\frac{500(501)}{2} = 500 + (500)^2$

$= 500 + 250000 = 250,500$

D.

20. Which of the following is true of the function $n\sqrt{n}$?

[A] $n\sqrt{n} \in \omega(n \log_2 n)$.

[B] $n\sqrt{n} \in \Theta(n \log_2 n)$.

[C] $n\sqrt{n} \in O(n \log_2 n)$.

[D] $n\sqrt{n} \in o(n \log_2 n)$.

[E] None of the above.

$\frac{n\sqrt{n}}{n\log_2 n} = \frac{\sqrt{n}}{\log_2 n} \to \infty$

$\therefore$ $n\sqrt{n} > n\log_2 n$
$n\sqrt{n} \in \omega(n \log n)$

A