

QUIZ 2: 60 Minutes

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Answer **ALL** questions.

NO COLLABORATION or electronic devices. Any violations result in an F.

NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.

10 points for each correct answer

You **MUST** show **CORRECT** work to
get credit.

When in doubt, **TINKER**.

Total
200

1. You have boys A, B, C, D and girls w, x, y, z . How many teams of 3 kids have both sexes.

☐ A 16.

☐ B 24.

☒ C 48.

☐ D 56.

☐ E None of the above.

$$\begin{aligned} 2B, 1G &\rightarrow \binom{4}{2} \times \binom{4}{1} = 6 \times 4 = 24 \\ 2G, 1B &\rightarrow \binom{4}{1} \times \binom{4}{2} = 4 \times 6 = 24 \\ \hline &148 \end{aligned}$$

2. How many subsets of $\{a, b, c, d, e\}$ contain a or b ?

☐ A 16.

☒ B 24.

☐ C 48.

☐ D 56.

☐ E None of the above.

$$\begin{aligned} a &\rightarrow 2^4 \\ b &\rightarrow 2^4 \\ a \cap b &\rightarrow 2^3 \end{aligned} \left. \vphantom{\begin{aligned} a &\rightarrow 2^4 \\ b &\rightarrow 2^4 \\ a \cap b &\rightarrow 2^3 \end{aligned}} \right\} \text{Inclusion-Exclusion: } 2^4 + 2^4 - 2^3 \\ = 16 + 16 - 8 \\ = \boxed{24}$$

3. You have 100 boys. 80 are young, 60 are fast. How many are neither young nor fast?

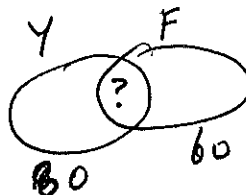
☐ A 10.

☐ B 20.

☐ C 30.

☐ D 40.

☒ E None of the above, or we don't have enough information.



4. You have 100 boys. 80 are young, 60 are fast, 50 are young and fast. How many are neither young nor fast?

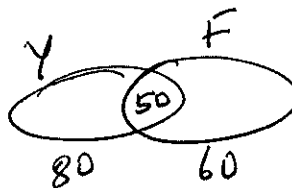
☒ A 10.

☐ B 20.

☐ C 30.

☐ D 40.

☐ E None of the above, or we don't have enough information.



$$\begin{aligned} \therefore Y \cup F &= 80 + 60 - 50 \\ &= 90 \\ \overline{Y \cup F} &= 100 - 90 \\ &= \boxed{10} \end{aligned}$$

5. Randomly pick a number from $\{1, 2, \dots, 100\}$ (each number has the same probability). What are the chances to pick a number divisible by 2 or 5?

☐ A 40%.

☐ B 50%.

☒ C 60%.

☐ D 70%.

☐ E None of the above.

$$\begin{aligned} \text{div } 2 &\rightarrow \frac{100}{2} = 50 \\ \text{div } 5 &\rightarrow \frac{100}{5} = 20 \\ \text{div } 5 \text{ and } 2 &\rightarrow \text{div } 10 \rightarrow \frac{100}{10} = 10 \\ \therefore P(\text{div } 5 \cup 2) &= \frac{50 + 20 - 10}{100} = \boxed{0.6} \end{aligned}$$

6. A 4-sided die is not fair. It rolls $\{1, 2, 3, 4\}$ with probabilities $\{x, 2x, 3x, 4x\}$. What is x ?

- ☐ A 0.05.
☒ B 0.10.
☐ C 0.15.
☐ D 0.20.
☐ E None of the above.

$$\begin{aligned} 10x &= 1 \\ \rightarrow x &= \frac{1}{10} \end{aligned}$$

7. You roll the die in problem 6. What are the chances the roll is 4 if the roll is at least 2?

- ☐ A $4/7$.
☐ B $4/8$.
☒ C $4/9$.
☐ D $4/10$.
☐ E None of the above.

$$\begin{aligned} P[4|2] &= \frac{P[4 \cap \geq 2]}{P[\geq 2]} = \frac{4/10}{2+3+4} \\ &= \frac{4/10}{9} = \frac{4}{9} \end{aligned}$$

8. For the die in problem 6, what is the expected value of a roll?

- ☐ A 1.5.
☐ B 2.0.
☐ C 2.5.
☒ D 3.0.
☐ E None of the above.

$$\begin{aligned} E[X] &= \sum x \cdot P(x) = 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{3}{10} + 4 \cdot \frac{4}{10} \\ &= \frac{1^2 + 2^2 + 3^2 + 4^2}{10} = \frac{1+4+9+16}{10} \\ &= \frac{30}{10} = 3 \end{aligned}$$

9. Chances of rain are 20%. Chances of heavy traffic in rain are 75%, but 25% in no rain. In heavy traffic, I'm always late. In light traffic I'm late 20% of the time. I was late. What are the chances it was raining?

- ☐ A $1/2$.
☒ B $1/3$.
☐ C $1/4$.
☐ D $1/5$.
☐ E None of the above.

$$\begin{aligned} &\begin{array}{l} \frac{1}{5} \text{ rain} \begin{cases} \frac{3}{4} \text{ heavy} \rightarrow \frac{1}{100} \text{ late} \\ \frac{1}{4} \text{ not heavy} \rightarrow \frac{4}{100} \text{ not late} \end{cases} \\ \frac{4}{5} \text{ no rain} \begin{cases} \frac{1}{4} \text{ heavy} \rightarrow \frac{1}{20} \text{ late} \\ \frac{3}{4} \text{ not heavy} \rightarrow \frac{12}{100} \text{ not late} \end{cases} \end{array} \\ &P[\text{Rain} | \text{Late}] = \frac{P[\text{Rain} \cap \text{Late}]}{P[\text{Late}]} = \frac{\frac{1}{100}}{\frac{3}{20} + \frac{1}{100}} = \frac{\frac{1}{100}}{\frac{16}{100}} = \frac{1}{16} \end{aligned}$$

10. In Problem 9, I was not late. What are the chances it was raining?

- ☐ A $1/2$.
☐ B $2/3$.
☐ C $3/4$.
☐ D $4/5$.
☒ E None of the above.

$$\begin{aligned} P[\text{Rain} | \text{not Late}] &= \frac{P[\text{Rain} \cap \text{Not Late}]}{P[\text{Not Late}]} \\ &= \frac{\frac{4}{100}}{\frac{52}{100}} = \frac{4}{52} = \frac{1}{13} \end{aligned}$$

11. Boys and girls are equally likely. A couple has 3 kids. What is the probability they have at least 2 boys?

☐ A 1/8.

☐ B 3/8.

☐ C 5/8.

☐ D 7/8.

☒ E None of the above

≥ 2 Boys: $\left. \begin{array}{l} GBB \\ GGB \\ BGB \\ BGG \end{array} \right\} 4 \times \frac{1}{8} = \boxed{\frac{4}{8}} = \frac{1}{2}$

12. What is the probability the couple in Problem 11 has at least 2 boys if you know they have at least 1 boy?

☐ A 3/7.

☒ B 4/7.

☐ C 3/8.

☐ D 4/8.

☐ E None of the above

$P[\text{at least 2 boys}] = 1 - P[GGG] = \frac{7}{8}$
 $P[2B | \geq 1B] = \frac{P[2B \cap \geq 1B]}{P[\geq 1B]} = \frac{4/8}{7/8} = \boxed{\frac{4}{7}}$

13. What is the probability the couple in Problem 11 has at least 2 boys if you know they have at least 1 girl?

☒ A 3/7.

☐ B 4/7.

☐ C 3/8.

☐ D 4/8.

☐ E None of the above

$P[\geq 1G] = 1 - P[BBB] = \frac{7}{8}$
 $P[2B \cap \geq 1G] = P[GBB, BGB, BGG] = \frac{3}{8}$
 $P[2B \cap \geq 1G | \geq 1G] = \frac{3/8}{7/8} = \boxed{\frac{3}{7}}$

14. A box has 6 fair, 2 two-headed, and 2 two-tailed coins. Flip a random coin. Compute $P[H]$?

☐ A 4/10.

☒ B 5/10.

☐ C 6/10.

☐ D 7/10.

☐ E None of the above

$P[H] = \underbrace{P[H|fair]}_{\frac{1}{2}} \underbrace{P[fair]}_{\frac{6}{10}} + \underbrace{P[H|2-headed]}_1 \underbrace{P[2-headed]}_{\frac{2}{10}} + \underbrace{P[H|2-tail]}_0 \underbrace{P[2-tail]}_{\frac{2}{10}}$
 $= \frac{1}{2} \cdot \frac{6}{10} + 1 \cdot \frac{2}{10} = \frac{5}{10} = \underline{\underline{\frac{1}{2}}}$

15. X and Y are random variables, with $E[X] = 2$ and $E[Y] = 3$. What is $E[3X + 4Y]$?

☐ A 5.

☐ B 15.

☒ C 18.

☐ D 35.

☐ E None of the above or not enough information.

Linearity. $3E[X] + 4E[Y]$
 $= 3 \cdot 2 + 4 \cdot 3 = 6 + 12 = \boxed{18}$

16. A random dart hits the bulls-eye 20% of the time. How likely is exactly 2 bulls-eyes in 5 random darts?

☐ A $320/5^4$.

☒ B $640/5^5$.

☐ C $320/5^4$.

☐ D $640/5^5$.

☐ E None of the above.

$p = \frac{1}{5}$ $P[k=2] = \binom{5}{2} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^3$
 $= 10 \times \frac{4^3}{5^5}$ $4^3 = 64$
 $= \boxed{\frac{640}{5^5}}$

17. In problem 16, what's the expected number of bulls-eyes in 5 random darts?

☒ A 1.

☐ B 1.5.

☐ C 2.

☐ D 2.5.

☐ E None of the above.

$E[k] = np = 5 \times \frac{1}{5} = \boxed{1}$

18. Bob has kids till a boy. Joe has kids till a girl. Girls are 3-times as likely as boys. What is the expected number of kids Bob and Joe have in total?

☐ A $2\frac{1}{2}$.

☒ B $5\frac{1}{3}$.

☐ C $6\frac{1}{4}$.

☐ D $7\frac{1}{5}$.

☐ E None of the above.

$P[\text{Girl}] = \frac{3}{4}$ $P[\text{Boy}] = \frac{1}{4}$
 $Kids = Kids_B + Kids_J$
 $E[Kids] = E[Kids_B] + E[Kids_J]$
 $= 4 + \frac{4}{3}$
 $= 4 + 1\frac{1}{3} = \boxed{5\frac{1}{3}}$

19. [Hard]. You flip a fair coin until two heads in a row, HH. What is the probability you make 6 flips?

☐ A $4/64$.

☒ B $5/64$.

☐ C $6/64$.

☐ D $7/64$.

☐ E None of the above.

Let $Q(k) = P[k \text{ flips}]$: $Q(k) = P[k|HH]P[HH] + P[k|HT]P[HT] + P[k|T]P[T]$ $k \geq 3$
 $= 0 \cdot \frac{1}{4} + Q(k-2) \cdot \frac{1}{4} + Q(k-1) \cdot \frac{1}{2}$
 $\Rightarrow Q(k) = \frac{1}{2}Q(k-1) + \frac{1}{4}Q(k-2)$

$Q(1) = 0$
 $Q(2) = \frac{1}{4}$

k	1	2	3	4	5	6
Q	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16} + \frac{1}{32}$	$\frac{3}{64} + \frac{1}{32}$
					$\frac{3}{32}$	$\frac{5}{64}$

20. You flip a fair coin until two heads in a row, HH. What is the expected number of flips?

☐ A 4.

☐ B 5.

☒ C 6.

☐ D 7.

☐ E None of the above.

$E[k] = E[k|HH] \cdot \frac{1}{4} + E[k|HT] \cdot \frac{1}{4} + E[k|T] \cdot \frac{1}{2}$
 $= \frac{2}{4} + (2 + E[k]) \cdot \frac{1}{4} + (1 + E[k]) \cdot \frac{1}{2}$
 $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + E[k] \cdot \frac{3}{4}$
 $\Rightarrow \frac{E[k]}{4} = \frac{3}{2} \Rightarrow \boxed{E[k] = 6}$

SCRATCH