11.1 k-Connected Graphs

We can now further extend a few of the concepts we discussed with restriction to 2-connected and 2-edge-connected to k-connected and k-edge-connected graphs. Given two vertices $x, y \in V(G)$, a set $S \subseteq V(G) - \{x, y\}$ is an x, y-**separator** if $G - S$ has no x, y-path. We define $\kappa(x, y)$ as the minimum cardinality over all possible x, y-separators and $\lambda(x, y)$ as the maximum cardinality over all possible sets of internally disjoint x, y-paths. Since any x, y-separator must contain an internal vertex of every internally disjoint x, y-path, we have $\kappa(x, y) \geq \lambda(x, y)$.

What follows is a generalization of Whitney's Theorem. **Menger's Theorem** states that for two vertices $x, y \in V(G)$ and $(x, y) \notin E(G)$ the minimum size of an x, y-separator equals the maximum number of pairwise internally disjoint x, y-paths; i.e, $\kappa(x, y) = \lambda(x, y)$. A graph is therefore k-**connected** if for all $x, y \in V(G)$, $\lambda(x, y) \geq k$.

We have similar concepts and terminology for k-edge-connectivity. Given two vertices $x, y \in V(G)$, a set $F \subseteq E(G)$ is an x, y-**disconnecting set** if $G - F$ has no x, y-path. We define $\kappa'(x, y)$ as the minimum cardinality over all possible x, y-disconnecting sets and $\lambda'(x, y)$ as the maximum cardinality over all possible sets of edge disjoint x, y-paths. A graph is k-**edge-connected** if for all $x, y \in V(G)$, $\lambda'(x, y) \geq k$. Likewise, $\kappa'(x, y) = \lambda'(x, y)$.