21.1 Forbidden Subgraphs

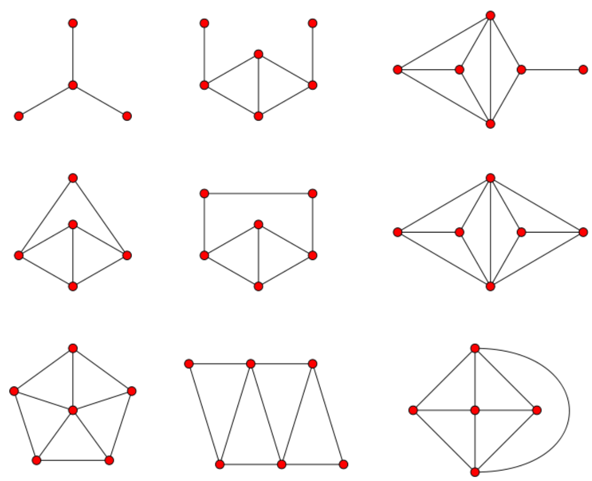
Consider the problems discussed yesterday with equivalent representations between G and line graph $L(G)$. Let's say that we wish to identify a maximum independent set on a general graph. Computing a maximum independent set is of exponential complexity, while a maximum match can be done in polynomial time. So, we can potentially simplify our problem if we're able to identify some graph H such that G is the line graph of H, or $L(H) = G$. If we can do that, then we can solve a maximum match on H and easily translate the solution to G.

Obviously, such an H is not going to exist for all graphs, otherwise that would imply we can solve a NP-hard problem in polynomial time. The question then becomes, for what conditions does there exist such a H? Below we're characterize some conditions we have of G such that a corresponding H exists.

For a simple graph G, there is a solution to $L(H) = G$ if and only if G decomposes into complete subgraphs, with each vertex of G appearing in at most two of these complete subgraphs.

A **double triangle** is an induced subgraph of graph G that consists of two triangles sharing an edge and no edge existing between the vertices that comprise the third vertex of each triangle. A triangle T is **odd** if $\exists v \in V(G) : |N(v) \cap V(T)|$ is odd. For a simple graph G, there is a solution to $L(H) = G$ if and only if G is claw-free and no double triangle of G has two odd triangles.

The graphs below list all **forbidden subgraphs**. For a simple graph G, there is a solution to $L(H) = G$ if and only if G does not contain any forbidden subgraph as an induced subgraph.



Being able to identify the graph H in $L(H) = G$ can be done in linear time. However, a discussion of such an algorithm is beyond the scope of the course.