## 10.1 Matching in General Graphs

For the most part, we've discussed matching restricted to bipartite graphs. We're going to generalize it now to all graphs. First define the function $o(G)$ as the number of **odd connected components** in $G$. As we know, odd connected components have an odd number of vertices. In order for a graph to have a perfect matching, we'll use what could loosely be considered as a generalization of Hall's Condition.

**Tutte's Theorem** states that a graph $G$ with a perfect match satisfies the inequality $\forall S \subseteq V(G) : o(G - S) \leq |S|$. Formally, a graph $G = (V, E)$ has a perfect matching if and only if for every possible vertex set $S \subseteq V(G)$, the subgraph induced by $V - S$ has at most $|S|$ connected components with an odd number of vertices. Let's develop a proof for Tutte's Theorem.

## 10.2 Independent Sets and Covers

A **vertex cover** of a graph $G$ is a set $Q \subseteq V(G)$ that contains at least one endpoint on all $e \in E(G)$. The vertices in $Q$ *cover* $E(G)$. An **edge cover** of $G$ is a set $L \subseteq E(G)$ such that $L$ has at least one edge incident on all $v \in V(G)$. The edges in $L$ *cover* $V(G)$.

The **König-Egerváry Theorem** states that if $G$ is a bipartite graph, then the size of a maximum matching in $G$ equals the minimum size of a vertex cover.

An **independent set** of vertices on a graph $G$ are a set of vertices that are not connected by an edge. The size of a maximum independent set on $G$ is called the **independence number** of $G$. For a bipartite graph, this isn't necessarily the size of the larger partite set.

In $G$, $S \subseteq V(G)$ is an independent set if and only if $\bar{S}$ is a vertex cover. Thus a maximum independent set is the complement of a minimum vertex cover, and their sizes summed equals the order of $G$.