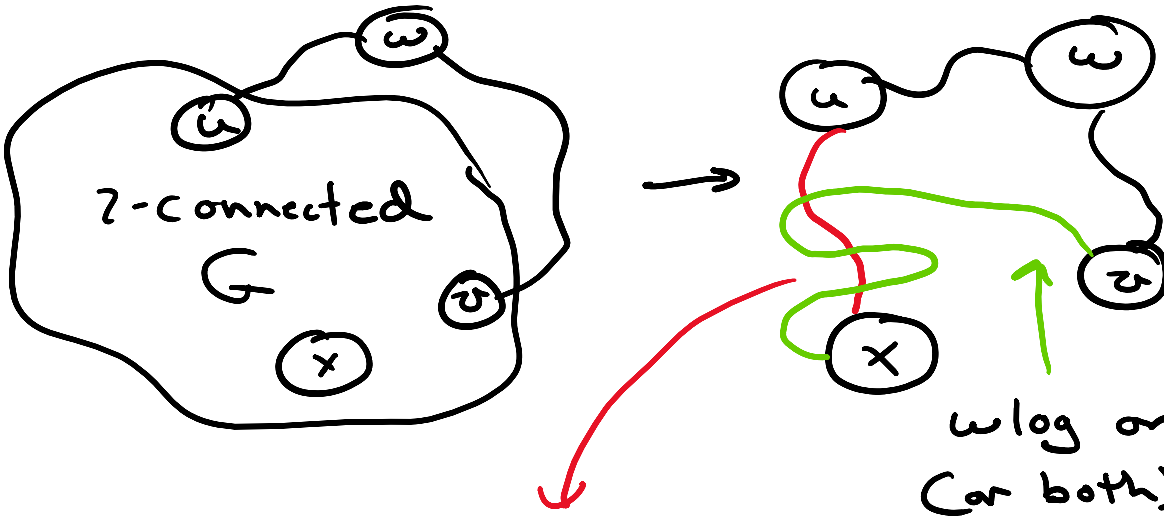
1 We talked about this during a couple proofs in class → Lecture 12 (but now w/o Whitney)



Do we have 2 w,x-ipds?

wlog one path (or both) might intersect the other (if not, we're done)

We know G is at least 2-connected → $\exists$ u,x & v,x paths

We originally demonstrated this using Whitney/Menger

Any other useful proof results?

Yes: our fan theorem

→ $\exists$ x, {u,v} fan as G is 2-connected

=> trivially, we combine disjoint ... with P to get idps □

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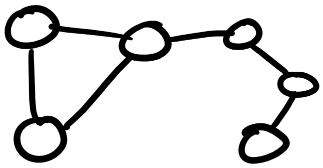
② 

$|V| = 6$
$\chi(G) = 3$
$\omega(G) = 3$

③ We need to show $\chi(H) = \omega(H)$ $\forall H \subseteq G$

Note: deleting vertices cannot create a chordless cycle, so

$\rightarrow \forall H \subseteq G : H$ is chordal)

SO: we only need to show that $\chi(G) = \omega(G)$ for chordal G
(any result applies to all $H \subseteq G$)

We know G has a simplicial elimination ordering $\{v_1 v_2 \dots v_n\}$

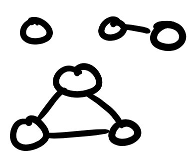
$\rightarrow$ For any v_i we have a K_m clique with v_i's $m-1$ neighbors in
$\{v_i \dots v_n\}$

with v_i's $m-1$ neighbors in
$\{v_1 \dots v_{i-1}\}$

$\rightarrow v_i$ gets color m in clique K_m

$\Rightarrow$ as this applies to all v_i, the largest clique created in G gives us the chromatic number and $\chi(G) = \omega(G)$ $\square$

(4) We prove for triangulations using our commonly-used inductive framework for planarity proofs

Basis:  $\rightarrow$ lines looking pretty straight

Assume we have $P(n)$ triangulation of our original G

$P(k) = P(n) - v \leftarrow v$ is our known $d(v) \leq 5$ vertex

I.H. on $P(k)$ gives us a straight line embedding line embedding

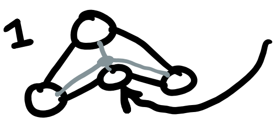
$P(k) \to P(n)$: consider v's face formed by $N(v)$

Case 1: Face is convex polygon

(includes $|N(v)|=3$) → we can place v anywhere and draw straight lines to all vertices

Case 2: $|N(v)| = 4$ and face is concave

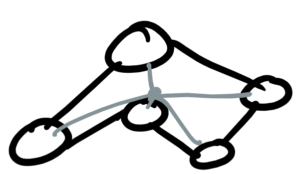
(at most 1 concave angle)



concave corner still has straight lines to all other corners, place v arbitrarily close to it

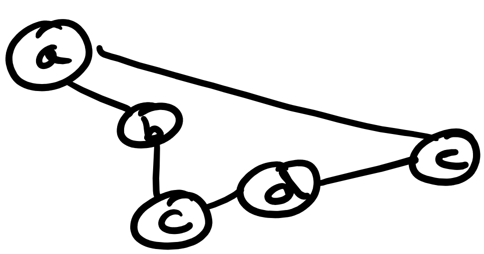
Case 3: $|N(v)| = 5$ and face is concave

Case 3a: only one concave corner



→ same as case 2, we can easily draw lines

Case 3b: 2 concave corners



Assume wlog that b, c are our concave corners

→ b has straight line to a, c, d

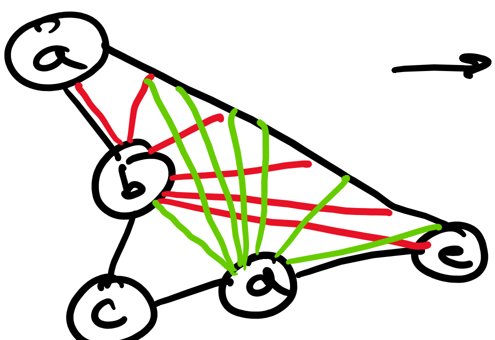
→ b has straight line to a, c, d

→ d has straight line to c, e, b

a c is convex

Q: is there a guarantee that b has no line to e AND d has no line to a?

A: No, consider wlog drawing a line from b→a, then moving the endpoint along (a, e)

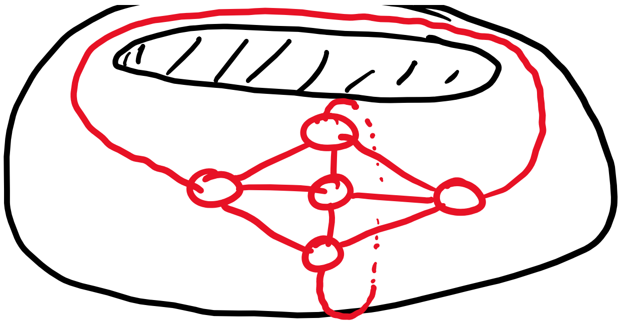


→ we can only not do this if we are blocked by d's corner

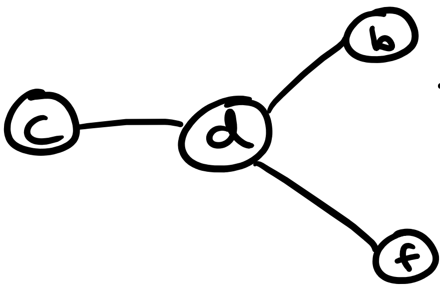
→ likewise, we observe the same with d and lines drawn on (e, a)

=> If both (b, e) & (d, a) lines are blocked, this implies no line (a, e) could exist contradiction □

(5) No: we can draw K_5 (or $K_{3,3}$) on a torus without crossings

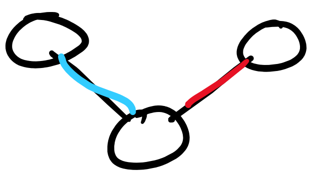


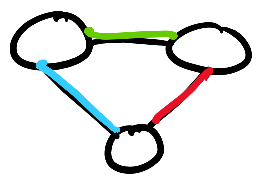
(6)



claw, which is a Forbidden subgraph, so no such H exists $\square$

(7)





$\chi'(G)=\Delta(G)$ $\chi'(H)=\Delta(H)+1$

(8) Hamiltonian for all $r \geq 2$

VIA ALGORITHM

in $K_{r,r}=G_{XY}$, label X, Y as

$x_1x_2\cdots x_r$, $y_1y_2\cdots y_r$

Our H.C.: for $i = 1 \dots (r-1)$:

$HC \leftarrow (x_i, y_i)$

$HC \leftarrow (y_i, x_{i+1})$

$HC \leftarrow (x_r, y_r)$

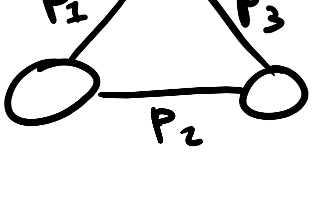
$HC \leftarrow (y_r, x_1)$

=> This constructs an H.C. for all $\boxed{r \geq 2}$

For an HP, the above applies minus the final step

=> so it can also work for $r = 1$ and therefore $\boxed{r \geq 1}$

9. As we've observed, a triangle forms with probability $(p_1 p_2 p_3)$ based on pairwise attachment probabilities



→ in CM graph: $p_{uv} = \frac{d(u)\,d(v)}{\ }$

→ in CM graph: $p_{uv} = \dfrac{d(u)\,d(v)}{2m}$

so for triangle $\{u, v, w\}$

$$P_{\triangle} = \frac{d(u)\,d(v)}{2m}\;\frac{d(u)\,d(w)}{2m}\;\frac{d(v)\,d(w)}{2m}$$

$$= \frac{d(u)^2\,d(v)^2\,d(w)^2}{(2m)^3}$$

And for unique triangles:
(DON'T OVERCOUNT!)

$$\#\triangle = \sum_{u<v<w}^{V(G)} \frac{d(u)^2\,d(v)^2\,d(w)^2}{(2m)^3}$$

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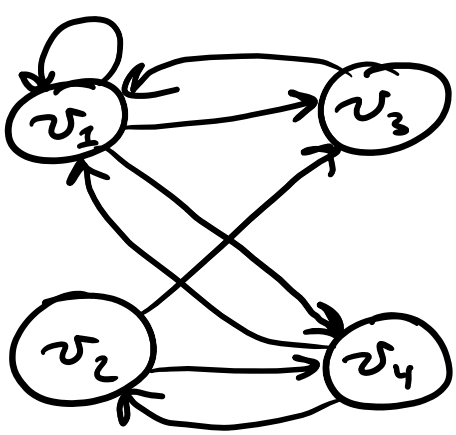

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

$$O^{-1} = \begin{bmatrix} 1/3 & & & \\ & 1/2 & & \\ & & 1 & \\ & & & \frac{1}{2} \end{bmatrix} \quad M = \begin{bmatrix} 1/3 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

(multiplied and took transpose in one operation)