**Graph Theory Final Practice Problems**

*Use these as study aids in conjunction with notes, homeworks, and weekly problems.*

1. Consider adding a new $u, v$-path $P$ to some 2-connected graph $G$, where $u, v \in V(G)$. Formally demonstrate that for any $w \in V(P)$, there exists two internally disjoint paths to any $x \in V(G)$. For this proof, you cannot use the results of Whitney's or Menger's Theorems.

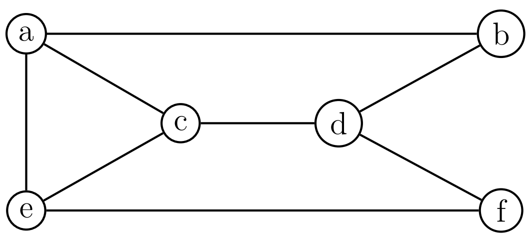
2. Draw a non-clique graph $G$ of at least 6 vertices where the chromatic number of $G$ is equal to upper bound given by Brooks' Theorem.

3. Prove that all chordal graphs are perfect.

4. A **straight-line embedding** of planar $G$ is a planar embedding where all edges are drawn as straight lines, instead of curves. Prove that all simple planar graphs have a straight-line embedding.

5. We observed that Kuratowski's theorem is valid for embeddings on 2D planes as well as spheres. Does it also hold for an embedding on a torus?

6. Prove or disprove whether the below graph $G$ is the line graph of some $H$, where $L(H) = G$.




7. Construct a connected and non-empty graph $G$ where $\chi'(G) = \Delta(G)$. Then, construct a graph $H$ where $\chi'(H) = (\Delta(H) + 1)$.

8. For what values of $r$ is biclique $K_{r,r}$ Hamiltonian? For what values of $r$ does $K_{r,r}$ have a Hamiltonian path? Prove your responses.

9. We have a configuration model with a degree sequence $S = \{d_1, d_2, \ldots d_n\}$. How many triangles can we expect in this graph?
   *Hint: You can present your answer as a summation or similar non-reduced form.*

10. We use the transpose of a transition probability matrix $M = (D^{-1}A)^T$ for algebraic PageRank computations. Create matrix $M$ using the following digraph $G$:
    $V = \{v_1, v_2, v_3, v_4\}$
    $E = \{e_1(v_1, v_1), e_2(v_1, v_3), e_3(v_1, v_4), e_4(v_2, v_3), e_5(v_2, v_4), e_6(v_3, v_1), e_7(v_4, v_2), e_8(v_4, v_1)\}$