

Graph Theory Homework 2

Due: 13 Feb 2026 at midnight EST as a PDF on Submittity

v1.1: Last Updated February 9, 2026

1. Prove correctness of Prim's Algorithm. (4 pts)
2. Consider graph G made by taking clique graph K_5 and deleting 2 edges that share no endpoints. Assign weights $W = \{1, 1, 2, 2, 3, 3, 4, 4\}$ to the edges of G in a way that guarantees a minimum spanning tree of G is unique. (4 pts)
3. Prove that in every tree T , any two maximum length paths of equal length must intersect (have a common vertex v). (4 pts)
4. As stated in class, weak induction is particularly use for proofs on trees, because we can generate all possible tree topologies by iteratively adding a single new leaf to an existing tree. Prove this. (Note: using weak induction here might result in circular logic, so be careful if you choose to go that route.) (4 pts)
5. Prove that the necessary condition for trees being maximally acyclic is also a sufficient condition. (4 pts)
6. Prove that all simple **connected** (v1.1) graphs with 4 or fewer vertices are graceful. (4 pts)
7. With Dijkstra's algorithm, we assumed non-negative edge weights on the input graph. Prove why this assumption is necessary. (4 pts)
8. Prove the following equivalence for simple connected graph G : G contains exactly one single cycle $\iff |E(G)| = |V(G)|$. (4 pts)