Graph Theory Homework 4

Due: 23 March 2026 at midnight EST as a PDF on Submitty

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1. Consider simple graph G. Prove that either G or its complement $\bar{G}$ must be connected. (4 pts)

2. Prove that G is connected iff for every partitiong of $V(G)$ into non-empty subsets V_1 and V_2, there exists some $e = (u, v) \in E(G) : u \in V_1, v \in V_2$. (4 pts)

3. Prove that a connected spanning subgraph of connected graph G contains all cut vertices and cut edges of G. (4 pts)

4. $e = (u, v)$ is a cut edge in G. Prove that at least one of u, v must be a cut vertex. Use this result to bound $\kappa(G)$ relative to $\kappa'(G)$. (4 pts)

5. Consider connected graph G with no even cycles. Prove that every block of G must be either an odd cycle or K_2. (4 pts)

6. Show that for connected graph G, $\forall v \in V(G) : d(v)$ is even iff for every maximal biconnected component $B_i \in G, \forall u \in V(B_i) : d(u)$ is even. *Hint: one direction is trivial and the other might require a bit of induction.* (4 pts)

7. G is a simple graph with $\Delta(G) \leq 3$. Prove that $\kappa(G) = \kappa'(G)$. (4 pts)

8. Consider bipartite graph B. Use Menger's Theorem prove that the size of a maximum matching is equal to the size of a minimum vertex cover on B. (4 pts)

9. Now, use the concept of network flows to prove that the size of a maximum matching is equal to the size of a minimum vertex cover on bipartite graph B. (4 pts)

10. Consider a standard s, t flow network N where every edge has unit capacity. On this network, we have a maximum flow of value k. Prove that we can remove any arbitrary $k - 1$ edges from N and still have at least one s, t-path. (4 pts)