Graph Theory Homework 5

Due: 3 April 2026 at midnight EST as a PDF on Submitty

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1. Prove that on some arbitrary G there will always exist some ordering of vertices such that greedy coloring will output an optimal solution. (4 pts)

2. Likewise, prove or disprove whether we can guarantee an ordering exists for some G where greedy coloring will output the worse-case solution. (4 pts)

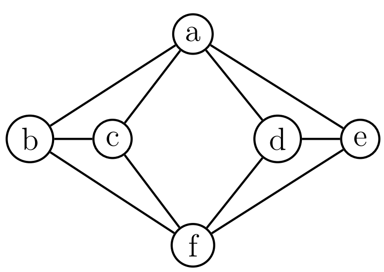
3. We have a vertex set V and two edge sets E_1 and E_2 defined on that set of vertices. We construct two graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ where, $\chi(G_1) = k_1$ and $\chi(G_2) = k_2$. Now, consider a third graph $H = (V, E_1 \cup E_2)$ where $\chi(H) = j$. Prove that $j \leq (k_1 \times k_2)$. (4 pts)

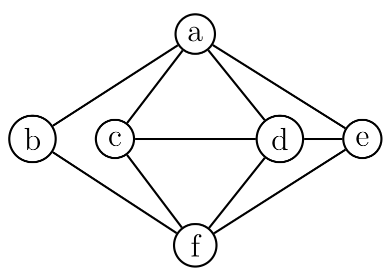
4. The independence number of some graph G is the size of the largest independent set, labeled as $\alpha(G)$. Prove that $\chi(G) \geq \frac{|V(G)|}{\alpha(G)}$. (4 pts)

5. We've defined a *claw graph* as the star graph S_4 or complete bipartite graph $K_{1,3}$. A claw-free graph is a graph without a claw. Prove that in a claw-free G the subgraph induced on the vertex sets of any two color classes will consist of paths and even cycles. (4 pts)

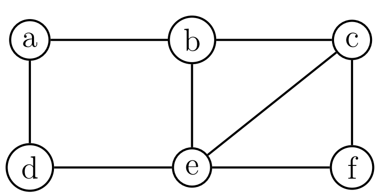
6. Claw-free G is k-colorable. Prove that there must exist a k-coloring on G where the cardinalities of the sets of vertices in each color class differ by at most 1. (4 pts)

7. Prove that the chromatic polynomials of the below 2 graphs are equal. (4 pts)





8. Consider the below G for the following problems.



   (a) Draw G' created via Mycielski's Construction on G. What is $\omega(G)$, $\omega(G')$, $\chi(G)$ and $\chi(G')$?

(b) Use the Fundamental Reduction Theorem to give the chromatic polynomial $\chi(G; k)$ of G. Use the polynomial to determine the chromatic number of G. (4 pts)

(c) Does the above G have a simplicial elimination ordering? Provide one if possible; if not possible, prove why not. What does that say about whether or not G is perfect? (4 pts)