

Graph Theory Homework 6

Due: 13 April 2026 at midnight EST as a PDF on Submittity

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1. Prove that the dual graph of any arbitrary connected planar G is connected. (4 pts)
2. G is outerplanar. Prove that $\chi(G) \leq 3$. (4 pts)
3. G is a connected triangulation with $|V(G)| \geq 3$. Prove the equivalence: $\chi(G) \leq 3$ iff $\forall v \in V(G) : d(v) = \text{even}$. (4 pts)
4. Prove that all planar graphs are a subgraph of some triangulation. (4 pts)
5. Prove that all triangulations are maximally planar. (4 pts)
6. G is 3-connected with $|V(G)| \geq 6$. Prove that if G contains a K_5 subdivision then G also contains a $K_{3,3}$ subdivision. (4 pts)
7. Draw a map that requires 4 colors. Then prove the dual graph of this map has a chromatic number of 4. (4 pts)
8. For each of the following, prove that G must be planar or give a counter-example.
 - (a) G has chromatic number $\chi(G) = 4$. (4 pts)
 - (b) G has $|V(G)| = 6$ and $|E(G)| = 11$. (4 pts)
 - (c) Every subgraph H of G where $H \not\cong G$ is planar. (4 pts)