Graph Theory Homework 7

Due: 24 April 2026 at midnight EST as a PDF on Submitty

v1.1: Last Updated April 20, 2026

1. Prove that every connected $L(G)$ with $|V(L(G))|$ = even has a perfect match. (4 pts)
2. Prove that bipartite graph G is a subgraph of some k-regular bipartite graph H where $\Delta(G) = k$. (4 pts)
3. Prove G is isomorphic to $L(G)$ iff G is ~~2-connected~~ (**v1.1**) 2-regular. (4 pts)
4. G is k-regular and has a cut vertex. Prove $\Delta(G) < \chi'(G)$. (4 pts)
5. For each of the following, prove that the described G must be Hamiltonian or give a counter-example.
   (a) The closure $C(G)$ of G is a clique. (4 pts)
   (b) G is biparte with sets X and Y and $|X| = |Y|$. (4 pts)
   (c) G is defined by $G = (G' + e)$, where G' has a Hamiltonian path. (4 pts)
   (d) G is the line graph of some H, where H is a non-Eulerian 2-connected graph. (4 pts)
6. How many triangles are expected in a $G(n, p)$ Erdös-Rényi graph in terms of n and p? (4 pts)
7. We have u and v in a $G(n, p)$ graph. Assuming p is sufficiently large to ensure the graph is connected, what is the expected shortest path distance between u and v in terms of n and p? (4 pts)