Graph Theory Midterm Tips

Common Mistakes and Proof Advice

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Outline

Good news! Mostly proofwriting issues and not graph theory material misunderstandings!

- Graph assumptions and edge cases
- Justifying statements and defining terms
- Don't do induction backwards
- Rapid fire misc tips

Graph assumptions and edge cases

Remember to consider if the graph is:

- Non-Simple
  - "graphic" does imply simple though
- Disconnected
  - Removing edges/vertices can disconnect the graph!

Justifying statements and defining terms

Nothing comes from nowhere! Introduce your terms and justify your statements.

- Where do terms come from?
  - Arbitrary instance to prove a goal (ex: "consider some graph, G, such that ...")
  - Proven existence (ex: "by Berge's we have some M-aug path, p")
  - Graph manipulation (ex: "$G' = G - e$")
- Statements should follow from some combination of previous statements, trivial observations, previously proven facts, and explanations tying them together.
- Make clear where everything comes from to hopefully catch simple mistakes

Don't do induction backwards

Start with a higher case goal, reduce to lower case(s), use that fact to prove your goal.

- Common format: "show $\forall G : p(G) \implies q(G)$"
  - ex: "G has no odd cycles $\implies$ G is bipartite"
- IH: $\left(\forall k < n : p(G_k) \implies q(G_k)\right) \implies \left(p(G_n) \implies q(G_n)\right)$
- Consider arbitrary G_n such that $p(G_n)$, we want to show $q(G_n)$
- Construct $G_k = f(G_n)$ such that $p(G_k)$ and $k < n$.
- By IH, $q(G_k)$. Show that implies $q(G_n)$.
- By induction we've shown $p(G) \implies q(G)$ for arbitrary G and therefore for all.

Don't do induction backwards (doing it backwards)

Doing it backwards, you don't necessarily show it for all:

- Consider arbitrary G_n such that $p(G_n)$ and therefore $q(G_n)$.
- Construct $G_{n+k} = f(G_n)$ such that $p(G_{n+k})$ where $n + k > n$.
- Show that $q(G_n)$ implies $q(G_{n+k})$.

We've only shown that $p(G) \implies q(G)$ for graphs constructable via $f^n(Base)$. Would need an additional (likely clunky) argument to show that this generalizes to all graphs.

Rapid fire misc tips

- Draw graphs! Especially for counterexamples. Quicker and clearer than describing with words.
- Prove everything the problem is asking for
    - Common issue on hw1 q2 (not showing that n is smallest) and hw2 q1 (not showing the result is a spanning tree)
- Prioritize presenting clear and correct logic over a Pretty Proof (you don't need to be verbose)

More Midterm Stuff

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How to prepare:

→ Understand *all* proofs thus far

→ practice more → (!) questions in book

→ plenty online

(other GT classes)

→ AI (ask for questions and solutions)

Exam approach

→ questions vary in difficulty / length (but weighted same)

→ do easy ones first

→ Usually, a straightforward response in mind → proof approach (parity, induction, extremal, etc.)

→ Other times, brute force is the only way of proof approachs

→ Draw it out → configurations

→ induction, parity, extremal arguments

all likely to show up

→ (just use strong induction)

→ can use many things for our induction variable

→ Questions cover all topics

→ usually, at least one question per lecture

→ and proof techniques