Graph Theory Midterm: Stuff to Know

General Info

- Exam will be in-class Thursday, February 26th. DSS students with extra time should plan on starting earlier.
- Material will cover Chapters 1-3 of the book AND any additional material and algorithms covered in class. This include Lectures 1–10, Homeworks 1–3 and WPS 1–5.
- Exam will be closed notes, closed book, closed neighbor.
- However, a crib sheet will be provided with common definitions, notation, and algorithms. Scratch paper will also be included. **No other outside material or scratch sheets are allowed.**
- Questions will require direct knowledge of the definitions and graph properties we discussed, applying algorithms and knowledge to problems, and working through proofs.
- Expect the exam to be of approximate difficulty and length of a homework assignment.

The following material is only a guide. Regardless of what is listed below, everything discussed in class and in the online notes can appear on the test unless explicitly stated otherwise.

Chapter 1

1. Basic graph definitions/classes - terminology, every term in bold in the notes; what makes a graph simple; what are complete graphs, bipartite graphs
2. Graph representation - adjacency matrices
3. Walks, cycles, paths, trails, etc. - definitions and differences; usage in proofs
4. Isomorphism - properties of isomorphic graphs; isomorphic classes; demonstrating isomorphism; automorphism class properties and enumeration
5. Eulerian graphs - directed and undirected properties and proofs
6. Graphic sequences - how to verify an integer sequence is graphic; create a graph using a given sequence

7. Connectivity - undirected connectivity properties

Chapter 2

1. Trees - basic properties; Cayley's formula; Prüfer Codes and algorithms, enumerative properties
2. Distances - diameter; radius; eccentricity; center
3. Spanning Trees - counting; edge contraction method
4. Graceful Labeling - what makes a graph graceful; what kinds of graphs are graceful
5. Minimum Spanning Tree - Krushkal's and Prim's algorithms
6. Shortest Paths - Dijkstra's Algorithm

Chapter 3

1. Matching - perfect/maximal/maximum; Berge's Theorem; Hall's Condition; Tutte's Condition; symmetric differences; M-augmenting and M-alternating paths
2. Covers - vertex and edge covers; independent sets and independence number; König-Egerváry Theorem, dominating sets
3. Maximum Bipartite Matching - Augmenting Paths Algorithm
4. Matching variants - maximum weight matching and stable matching

Other Material

1. Proof techniques - weak/strong induction; necessity and sufficiency; extremal arguments; algorithmic proofs; general structural arguments on graphs; see the bag o' tricks document from earlier in class