WP2 - Solution

Wednesday, January 28, 2026 4:55 PM

$\mathcal{G} = \{$connected, acyclic, non-null, simple$\}$

Basis: $P(0) = 0$ ← single vertex

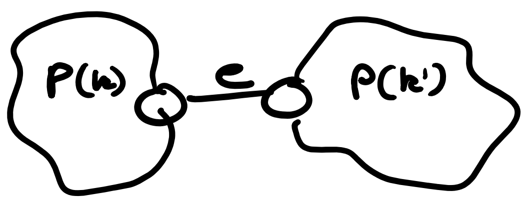
↖ induction on $|E|$

Assume we have $P(n) \in \mathcal{G}$

we want to show $|E(P(n))| = |V(P(n))| - 1$

Select any $e \in E(P(n))$

Consider $P(n) - e$



$P(n) - e \Rightarrow P(k)$ and $P(k')$

$\rightarrow P(k), P(k') \in \mathcal{G}$ ✓

as edge deletion will create two components, within each component edge deletion does not affect any of $\{$simple, connected, acyclic, non-null$\}$

Via I.H. (induction hypothesis)

$|E(P(k))| = |V(P(k))| - 1$

$$|E(P(k))| = |V(P(k))| - 1$$
$$|E(P(k'))| = |V(P(k'))| - 1$$

Note: $|V(P(n))| = |V(P(k))| + |V(P(k'))|$

$$|E(P(n))| = |E(P(k))| + |E(P(k'))| + 1$$

for e

$$|E(P(k))| + |E(P(k'))| = |E(P(n))| - 1$$

$$|E(P(k))| + |E(P(k'))| = |V(P(k))| - 1 + |V(P(k'))| - 1$$

$$|E(P(n))| - 1 = [|V(P(k))| - 1] + [|V(P(k'))| - 1]$$

$$|E(P(n))| = \underbrace{|V(P(k))| + |V(P(k'))|}\ \underbrace{-2 + 1}$$

$$\boxed{|E(P(n))| = |V(P(n))| - 1} \ \square$$