(1) We do the construction described in class to turn undirected G into directed flow network N

For any $x \in V(G)$: x is source $s \in V(N)$

For any $y \in V(G)$: y is sink $t \in V(N)$

all other vertices in $V(G) \rightarrow V(N)$

$\forall e = (u,v) \in E(G)$: $\{(u,v),(v,u)\} \in E(N)$

directed edges with infinite capacity

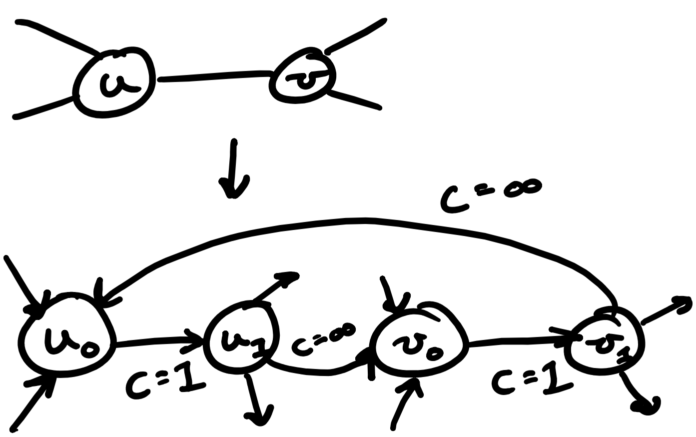
We further transform N to make each unit of flow following disjoint paths

$\forall v \in V(N) \xrightarrow{\text{replace}} \{v_0, v_1\} \in V(N)$

(except s,t)

$(v_0, v_1) \in E(N)$

with capacity 1



v_0 gets all in edges of v

v_1 gets all out edges of v

We now compute max flow f

↳ this equivalently gives us min cut

Note: only unit capacity $(i_0 \to i_1)$ edges will be on the cut

↳ this will equivalently give us a vertex cut on G

We next consider following a single unit of flow from s to t

↳ only a single unit can go through a single $(i_0 \to i_1)$ edge and each unit follows some s,t-path

→ each s,t-path will be equivalently vertex disjoint on G

→ the total flow and therefore number of s,t-paths will be equal to the cut

$\Rightarrow$ so we have $\underset{\text{on } N}{\lambda(s,t)} = \underset{\text{on } G}{\lambda(x,y)}$ disjoint so we have $\underset{\text{on } N}{\lambda(s,t)} = \underset{\text{on } G}{\lambda(x,y)}$ disjoint

paths and $\underset{\text{on } N}{K'(s,t)} = \underset{\text{on } G}{K(x,y)}$ cuts,

where $K'(s,t) = \lambda(s,t) = \lambda(x,y) = K(x,y)$

$\square$