

① $k=2 \Rightarrow$ the graph has at least a nontrivial path

\Rightarrow we know from class that graph with at least $\chi(G) \geq 2$ must have at least one edge ✓

$k=3 \Rightarrow$ the graph must have at least one cycle

\Rightarrow as acyclic graphs are ^{$(\chi(G)=2)$} bipartite, any graph with $\chi(G) \geq 3$ must have a cycle in it ✓

② In class, we saw that k -critical graphs are at least $(k-1)$ -edge-connected

\rightarrow we know $\chi'(G) \leq \delta(G)$ from before

$\Rightarrow (k-1) \leq \delta(G)$