WP10 - Solution

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Using: $\chi(G) \leq 4$ for planar G

1. Recall our construction that showed we can trivially add a new vertex and connect it to all vertices in an outerplanar graph

Result is planar

must be colored with unique color

→ the new vertex will increase the chromatic number exactly by 1 and result is still planar

→ We know all $\chi(G) \leq 4$ for a planar G

$\Rightarrow$ So any outerplanar G has $\chi(G) \leq 3$ □

has $\chi(G) \leq 5$ $\square$

(2) We will classify edges based on colors of endpoints

- Assume colors $\in \{1, 2, 3, 4\}$

H_1: assign all edges where endpoints have the same parity

H_2: assign edges with differing parity

Note: any path in H_1 will alternate

$0 \to 2 \to 0 \to 2 \to$ etc.

OR $1 \to 3 \to 1 \to 3 \to$ etc.

AND: any path in H_2 will alternate

(even $\to$) odd $\to$ even $\to$ odd $\to$ even $\to$ etc.

$\to$ Any closed path must be of even length in H_1 & H_2

$\Rightarrow H_1$ & H_2 have no odd cycles and are therefore bipartite $\square$