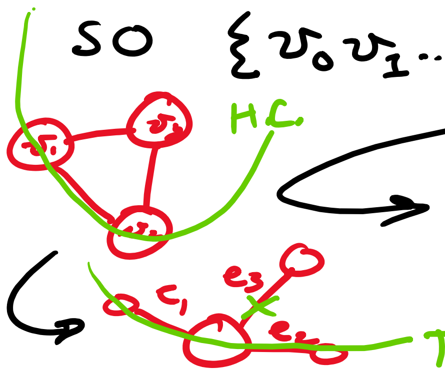
WP11 - Solution

Tuesday, April 21, 2026 1:11 PM

(⇒) Consider the H.C. on $L(G)$ as a traversal of $V(L(G)) \rightarrow \{v_0 v_1 \dots v_n v_0\}$

→ These vertices are the edges of G per the definition of line graphs

so $\{v_0 v_1 \dots v_n v_0\} \longrightarrow \underbrace{\{e_0 e_1 \dots e_n e_0\}}$



→ If it is possible to traverse edges in this order → we are done

Otherwise: we have some e_i incident on a vertex shared with e_{i-1} and e_{i+1}

→ we remove e_i but it is still covered

⇒ we have a closed tour that is trivially a cover ✓

(⇐) We construct a H.C. on $L(G)$

ALGORITHMICALLY

Consider our closed trail on G

$T = \{e_0 e_1 \dots e_i e_0\}$

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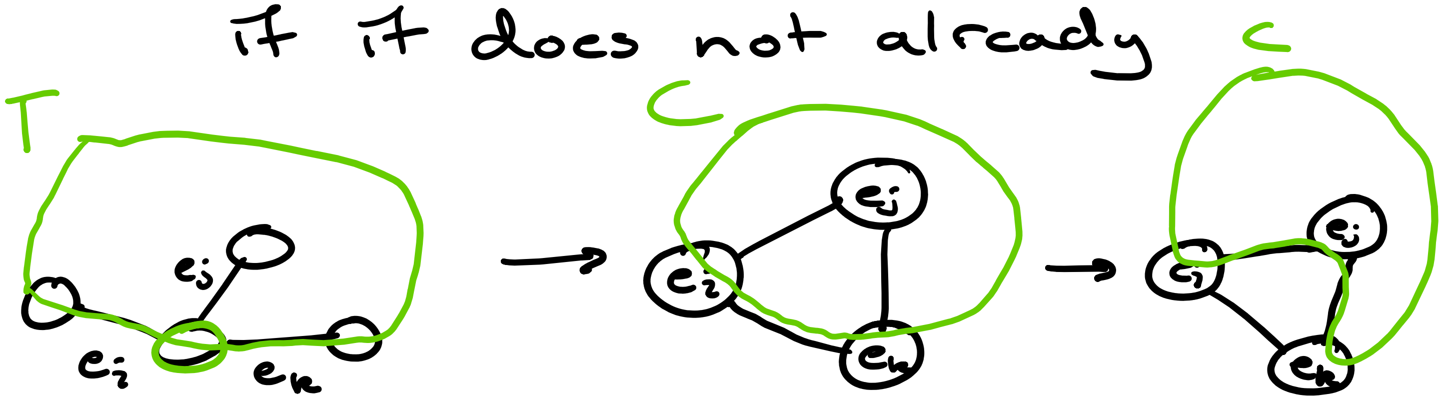
Note: this does not (necessarily) contain all edges of G

BUT: it contains a set of verts that are incident to all edges

Directly translate T to a cycle on $L(G)$, and consider expanding the cycle to contain all $V(L(G))$ if it does not already



As any e_j not in T is incident on a shared vertex in $V(T)$, it is part of a clique with some e_i, e_k in $L(G)$

→ we can trivially add e_j to C via the edges in that clique

→ if there are multiple e_j on the same vertex, we can also iterate this process

⇒ we will eventually add all e_j to C, and since $V(T)$ is a cover, C will include all of $V(L(G))$ and therefore be a H.C. $\square$