Graph Theory Weekly Problems 12
Due: 23 April 2026 at midnight EST as a PDF on Submitty
v1.0: Last Updated April 22, 2026

1. You and your crew of Dirty Dave, Bad Boi Brian, Carlitos, and Bone Saw Eddie (aka D-squared, B-cubed, C-prime, and BS-Extreme) compete with each other in a Yu-Gi-Oh! Trading Card Game™ tournament, with the following outcomes:

   - You defeat Dirty Dave and Bad Boi Brian
   - Bone Saw Eddie defeats Dirty Dave and Carlitos
   - Bad Boi Brian defeats Bone Saw Eddie
   - Carlitos defeats you
   - Dirty Dave defeats Carlitos

   Because you and Bone Saw Eddie have the same number of victories, you both agree that obviously the overall winner should be determined as the one with the highest PageRank calculated on a graph modeling the competition.

   (a) First, create this graphical model where each competitor is a vertex and an edge exists between any two competitors that competed. *Orient* the graph so that each edge points to the victor.

   (b) Compute the transition probability matrix M as $M = (D^{-1}A)^T$, where A is the adjacency matrix of the graph and D is a diagonal degree matrix.

   (c) Initialize PageRanks equally among competitors and calculate a single iteration using the simple algebraic formulation. Who wins?