

# Stabilizing Markets via a Novel Auction Based Pricing Mechanism for Short-term Contracts for Network Services

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## Abstract

This paper studies pricing mechanisms for short-term contracts for network services in a recurring auction with sealed bids. We propose and evaluate a novel winner selection policy in such an auction. The new approach was motivated by an observation that in a recurring auction enough customers must be willing to participate in future auction rounds to prevent collapse of prices. Using simulations, we compare the proposed mechanism with traditional ones. The results demonstrate that the new method (i) increases revenues of the network service provider, (ii) minimizes loss of fairness, and (iii) enlarges the active customer base of a recurring auction.

## Keywords

Network Service, Pricing, Auction, Negotiation, Bidder Drop Control

## 1. Introduction

Several dynamic pricing schemes for selling network services have been proposed recently [9,10,11]. They constitute an essential component of dynamically adaptable service management frameworks for the sale and delivery of a wide range of network services. They have been proposed as an efficient solution for the congestion control, service admission control, fair allocation of network resources and revenue maximization. In such frameworks, prices help to control dynamically changing network environments. Most of the proposed dynamic pricing schemes use an auction [6,7,8] because of its ease of understanding by the customers and network service providers and its decentralized pricing scheme that eliminates the need for the seller to decide the exact price of products [3].

The precise modeling of a customer behavior is a critical part of validation of the designs for auction based dynamic pricing schemes [15]. However, the previously considered designs for auctioning network services disregarded recurring nature of those auctions. Generally, an allocation of a network service is made for a specific time only, and once the allocated resources become free, the network service provider needs to offer them to the customers again. Hence, an auction for network services should be regarded as a recurring one.

Using traditional auction mechanisms, such as English or Vickrey ones, in a recurring auction may result in an inevitable starvation of certain customers. As a result, the

affected customers may drop out of the future auction rounds, thereby decreasing the long-term demand for the traded products. Consequently, the network service provider cannot guarantee its revenues. The conclusion is that to stabilize revenues in a recurring auction, the network service provider must prevent price collapse that requires controlling the supply of resources and solving the *customer drop problem*.

Although a lot of attention was paid to controlling resource supplies, to the best of our knowledge, the customer drop problem has not been addressed. In this paper, we propose a novel pricing mechanism in a recurring auction for short-term contracts for network service. It resolves the customer drop problem, eliminates resource waste and restores the symmetry of the negotiation power between the customers and the service provider that is broken in the traditional auction mechanisms. The proposed mechanism is applicable to a multiple winners, discriminatory pricing, and sealed bid auction with the seller reservation price.

The remainder of this paper is organized as follows. In section 2, we consider potential negotiation scenarios for network services and use them to analyze the problems motivating our approach. Section 3 illustrates the novel auction pricing mechanism. The proposed mechanism is verified by various experiments that are described (together with their results) in section 4. Finally, summaries of the contributions and of the future work are included in section 5.

## **2. Negotiation scenarios and motivations**

Different negotiation scenarios require different pricing mechanisms to efficiently contract the resource allocation [2]. Hence, a precise analysis of the potential negotiation scenarios is necessary to design an efficient auction pricing mechanism.

### **2.1 Potential negotiation scenarios**

From the market structure point of view, an auction for network services can be regarded as an operated market with one-to-many participants (i.e., one network service provider and many customers), and in which multiple units of homogeneous goods are traded. The latter are network resources for premium quality, homogeneous network services that are requested recurrently by the customers for a specific time interval. Auction winners make contracts with a network service provider at the price that they bid in the auction.

We assume that the traded services are inelastic network applications, such as real-time voice and video applications that require the fixed amount of network resources to achieve the adequate quality of service (QoS). For this reason, the predetermined part of network resources is allocated to the traded services. The size of this part, assumed here to be constant for all auction rounds, defines the upper limit on the number of winners in each auction round.

Our negotiation scenario is based on two types of contracts. Long-term contracts are used for network services with the best-effort quality whereas short-term contracts are awarded via an auction and apply to the guaranteed premium quality network services. Our negotiation scenario focuses on recurring, short-term contracts only.

The quality of network services is dictated by the capacity of network resources (such as bandwidth, etc.) that are allocated to them. Since network resources assigned for a given time interval are perishable (that is, they perish if not used), we can characterize them as time sensitive goods. We make the followings assumptions throughout the paper: Each customer's true valuation (i.e., her willingness to pay) is restricted by her wealth that is unevenly distributed among customers. Thus, the true valuation of each customer is independently distributed (this is often called an Independent Private Value Model assumption). Each customer keeps his true valuation constant during the recurring auction. Additionally, customers are risk neutral and symmetric. Finally, unless clearly stated otherwise, we assume that customers who dropped out of an auction do no return later to it regardless of the winning prices.

## 2.2 Motivating problems

With the recurring demand for and time sensitivity of network resources, the traditional auction mechanisms, when applied to network services, cause the following problems.

**1) Asymmetric balance of negotiation power:** In most of the general auction mechanisms, the prices bid in an auction are dependent only on customer's willingness to pay for the traded goods. Hence, winning prices reflect intentions of only customers, but not of the network service provider.

**2) Resource waste:** To resolve the asymmetric balance of negotiation power problem, the seller's reservation price has been introduced into the general auction mechanism [4], leading to the so-called *seller's optimal auction*. Only bids higher than the seller's reservation price are considered during winner selection. However, in case of time sensitive goods, such as network services, the seller's reservation price causes resource waste. Resources unused because of the restriction on the number of winners imposed by the reservation price are wasted.

**3) Bidder drop problem:** Prices bid in an auction are dependent on the each customer true valuation of the traded goods. Each customer wealth influences such valuation that is an upper bound on customer's bids. An uneven wealth distribution can cause starvation of poor customers in a recurring auction. Most previous auction studies regarded the starvation as one of the methods for decreasing the customer's demand. However, a frequent starvation for the traded goods decreases the customer's interest in future auction rounds. If a customer concludes that it is impossible or unlikely that he will win at the price that he is willing to pay, he will drop from the future auction rounds. Such a drop out of an auction decreases the number of active customers in the future rounds. We will refer to the active customers as *bidders*.

A drop in the number of bidders gradually decreases the price competition. The remaining bidders constantly win and as a result they decrease their bidding prices for future auction rounds to maximize their expected profit. In the long run, the bidding price needed for the win may collapse to a very low level and revenues of the service provider may plummet to the unacceptable levels.

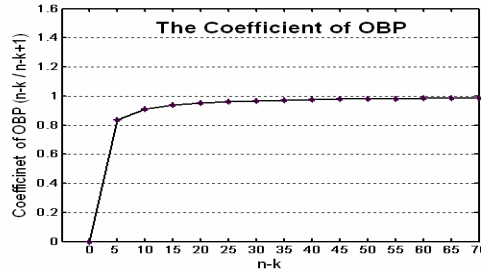
According to our assumptions, customers who participate in an auction for premium

quality network service already have entered a long-term contract for the best-effort quality network services. Thus, even if they lose in an auction, they always have an alternative (i.e., the best-effort quality network service) available for their requests. This is another important motivation for customers to drop out of the auction in case of starvation.

The importance of bidder drop control (i.e., maintaining a sufficiently high number of active customers)<sup>1</sup> in an auction can be shown by the following theoretical reasoning that is derived from the one made by McAfee and McMillan [3] to justify the first price sealed bid auction with a single winner. Under the assumptions of (i) a uniform distribution of bidder's private true valuations, and (ii) risk neutral bidders, the optimal bidding price (i.e., that is the price that optimizes the bidder expected profit) for bidder  $i$  in each auction round with discriminatory pricing, sealed bid with or without the reservation price is

$$b_i^* = \left(\frac{n-k}{n-k+1}\right) \cdot t_i, \quad (1)$$

where  $b_i^*$  denotes the optimal bidding price of bidder  $i$ ,  $t_i$  represents the private true valuation of bidder  $i$ ,  $n$  denotes the number of active bidders (i.e., participants in an auction), and  $k$  represents the number of possible winners in each auction round based on available resources. The proof of this formula is given in Appendix A.



**Figure 1:** A coefficient of the optimal bidding price

To maximize the seller's income from an auction, the optimal bidding price of each bidder should be close to his true valuation [2]. To avoid wasting of resources, the number of winners in each auction round should be constant. Hence, to keep the optimal bidding price from Equation (1) close to the true valuation, the number of bidders should be high. Figure 1 shows the change of coefficient  $(n-k)/(n-k+1)$  of the optimal bidding price as a function of the number of bidders (number of winners,  $k$ , is kept constant).

Our experimental results show that the three described above motivating problems indeed arise in our negotiation scenarios. As shown in Figure 2 (A), 28.6 % of

<sup>1</sup> In a Vickrey auction, theoretically, the optimal bidding price is independent of the number of participants. Yet, Vickrey himself noticed in his seminal work [13], that when the competition was weak, then the revenues from the Vickrey auction become small. Hence, the bidder drop control is also important in the Vickrey auction. Several other examples of similar phenomena were found in the real world situations reported in [14].

network resources are wasted by allocating resources only to the qualified customers in each auction with the reservation price.<sup>2</sup> Figure 2 (B) shows that the bidder drop rate in a recurring auction directly affects the average winning price (i.e., the revenue of the seller) in an auction with exponential distribution of the customer wealth.<sup>3</sup>

To solve the motivating problems, we propose a novel auction mechanism based on the Bidder Drop Control (BDC) winner policy described in the next section. This mechanism (i) maximizes the network service provider’s revenue, (ii) minimizes loss of fairness that is caused by the bidders dropping out of an auction, and (iii) enlarges the active customer base in a recurring auction.

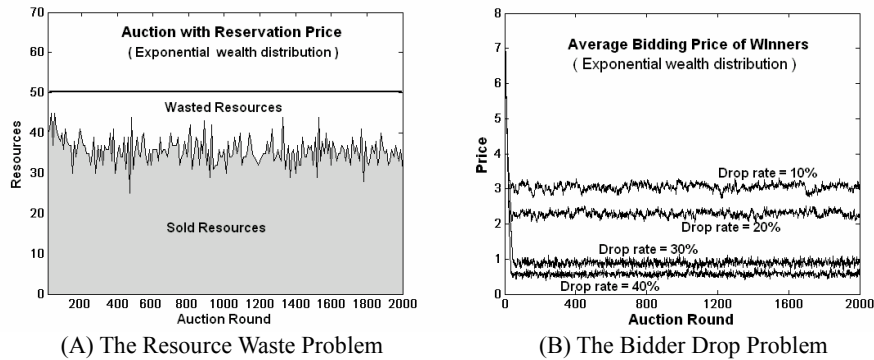


Figure 2: The resource waste and the bidder drop problem

### 3. Auction with BDC winner policy

#### 3.1 Illustration of main idea

The main idea of our paper is based on the demand-supply principle of microeconomics [1]. As shown in Figure 3, when the overall bid price decreases (i.e., the entire demand curve changes from D1 to D2) during a recurring auction, the minimum market clearing price drops from  $p_1$  to  $p_2$ . In such a case, to maintain the minimum market-clearing price at  $p_1$ , the service provider should decrease the supply of resources from  $q_1$  to  $q_2$  (i.e., the entire supply curve needs to be changed from S1 to S2). Inversely, when the overall bid price increases, the service provider may increase the supply. However, when the service provider decreases the supply of network resources for the given time period, the unsold resources are wasted. Thus, in the proposed auction mechanism, the “unsold” network resources ( $q_1$ – $q_2$  in Figure 3) are assigned to the bidders who have high probability of dropping out of the forthcoming auction round. This assignment prevents such a drop and keeps enough bidders in the auction to maintain the competition for resources.<sup>4</sup> Simultaneously,

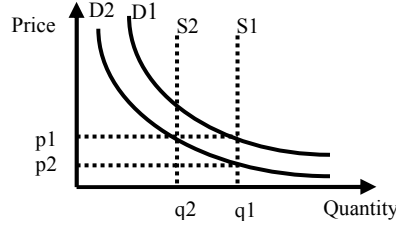
<sup>2</sup> This result is based on the experimental scenario described in Section 4.1 with the seller’s reservation price equal to 5.0, the exponential wealth distribution and no bidders dropping out of an auction. With bidders dropping, the waste of network resources will be even more pronounced than in the presented result.

<sup>3</sup> This result is also based on the experimental scenario described in Section 4.1.

<sup>4</sup> There are several previously studied auction mechanisms that focused on maximizing seller’s revenue, called seller’s optimal auction [4] [5]. However, they are based on trading non-perishable goods and do not consider the bidder drop problem in a recurring auction.

using “unsold” resources for bidder drop control resolves the resource waste problem and increases the number of winners. To implement the described above main ideas, we needed to answer the following two questions:

- (1) What is optimal number of network resources reserved for bidder drop control?
- (2) What is the efficient way of controlling the bidder drop problem?



**Figure 3:** The impact of the Demand-Supply principle

In the subsequent sections we provide answers to these questions. Here, we define the basic notions of bidders, bidding prices, and goods that are fundamental to the proposed auction mechanism.

- 1) **Bidders:** There are  $n+1$  bidders, denoted by  $i=0, \dots, n$ , including  $n$  customers,  $i=1..n$ , and a network service provider  $i=0$ . Each bidder enters her bidding price  $b_0, b_1, b_2, \dots, b_n$  in each auction round. We assume a sealed bidding, thus only a bidder and the network service provider can communicate.
- 2) **Goods:** There are  $R$  units of network resources that are assigned to the premium quality homogeneous network service for the predefined time period. The network service provider trades these assignments in each auction round. Each customer requires one unit of network resources for the premium quality network service.

### 3.2 Classification of customers

The first step in each auction round is to classify each customer based on his bidding price  $b_i$ , where  $i=1, \dots, n$ , and the network service provider's bidding price  $b_0$ . Two classes, Traditional Winner (TW) and Traditional Loser (TL), are based on the ascending rank of each customer's bidding price and the number of available resources; those customers will be the winners (losers, respectively) in a traditional auction. The numbers of customers in the TW class, denoted by  $N_{tw}$ , and the TL class, denoted by  $N_{tl}$ , are

$$N_{tw} = R; N_{tl} = n - N_{tw} = n - R \quad (2)$$

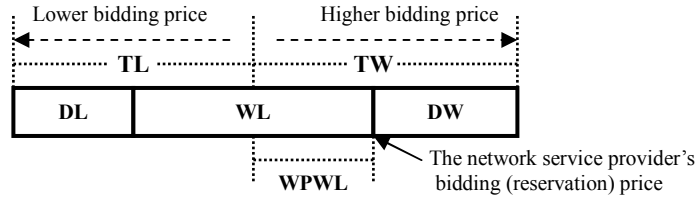
The network service provider classifies also customers into Definitely Winner (DW), Definitely Loser (DL), and Winner or Loser (WL) classes using the following conditions:

$$\begin{aligned}
i \in DW & \text{ if } b_i \geq b_0 \text{ \& } r_i > n - N_{pw}, \quad i = 1, 2, \dots, n. \\
i \in DL & \text{ if } b_i = 0, \quad i = 1, 2, \dots, n, \\
i \in WL & \text{ otherwise.}
\end{aligned} \tag{3}$$

where  $r_i$  denotes the rank of  $i$ -th bidder among all customers.

The DW class customers are winners without any additional considerations, since they bid prices higher than the network service provider did and there are enough resources to assign to all of them. The DL class represents the customers who already dropped out of the auction. The customers who are in the WL class can be winners or losers depending on the bidder drop control mechanism. The Winner Portion of Winner or Loser class (WPWL) is defined by the capacity of the network resources that are reserved for the bidder drop control mechanism. Thus, the number of customers  $N_{wpwl}$  who can be winners in the WL class is

$$N_{wpwl} = R - N_{dw} \tag{4}$$



**Figure 4:** A classification of customer's classes

### 3.3 Valuable Last Loser First (VLLF) algorithm

By definition, the bidder drop control mechanism applies only to customers in the WL class, so it must include an efficient strategy for selecting winners in this class. For this purpose, we propose the Valuable Last Loser First (VLLF) algorithm that consists of two phases. In the first phase, the VLLF algorithm selects customers in the WL class who have high probability of dropping out of the forthcoming auction round. The second phase of the VLLF algorithm compensates for the loss of fairness resulting from the first phase.

In the first phase of the VLLF algorithm, the bidders in the WL class who lost in the previous auction round but bid a higher price than in the previous auction round are marked as potential winners. The marked bidders are ranked according to their bidding price and up to  $N_{wpwl}$  highest ranked marked bidders are selected as winners.

If the number of the marked bidders is smaller than  $N_{wpwl}$ , the remaining resources are allocated in the second phase of the algorithm. The winner selection in the first phase is influenced by the bids in the previous auction round, so there could be some loss of fairness. To compensate for it, in the second phase, the highest bidding unmarked bidders in the WL class are selected as winners of the remaining resources. By marking only those last losers who bid higher in the current round than in the previous one, the algorithm prevents bidders with low bidding patterns from becoming winners.

### 3.4 Optimal service provider's bidding price (reservation price)

The bidding price of the service provider defines the minimum bid ensuring consideration for the membership in the DW class (so it plays the same role as the reservation price does in the Riley and Samuelson's optimal auction [4]). Finding the optimal bid for the service provider is equivalent to finding the optimal number of network resources to be reserved for the bidder drop control.

#### 3.4.1 Service provider's minimum cost

Let  $C_m$  denote the minimum cost of a unit of traded resources. This cost can also be interpreted as the service provider's desired minimum price for the unit of network resources. A specific mechanism for deciding  $C_m$  is beyond the scope of this paper.

#### 3.4.2 Optimal range of the service provider's bidding price

In this section, we derive a range for the optimal value of the reservation price. The revenue of an auction round with the VLLF bidder drop control algorithm should be larger than the network service provider's profitability revenue. The sufficient condition to ensure this constraint is:

$$b_0 \cdot N_{dw} + P_{wimp} \cdot (R - N_{dw}) > C_m \cdot R, \quad (5)$$

where  $P_{wimp}$  represents the minimum bidding price of winners in the WL class. Hence

$$P_{wimp} > \frac{C_m \cdot R - b_0 \cdot N_{dw}}{R - N_{dw}} \quad (6)$$

To control bidder drops efficiently,  $N_{dw}$  should be less than  $R$  to preserve some units of network resources for the VLLF bidder drop control algorithm. Hence, the following conditions on  $N_{dw}$ , and  $P_{wimp}$  can be derived:

$$0 \leq N_{dw} < R; \quad 0 < P_{wimp} < b_0 \quad (7)$$

Using inequalities (5) and (7), the condition for the optimal reservation price values is

$$b_0 > C_m \quad (8)$$

Therefore, the network service provider should bid a higher price than her minimum cost of a unit of network resources to maintain the profitability of each auction round.

The upper bound of a range for the optimal reservation price is constrained by the interrelationship between the three types of customer's classes and fairness. As shown in Table 1, an increase in the reservation price decreases the number of customers in the DW class (i.e.,  $N_{dw}$  decreases). This change results in an increase in the number of network resource units reserved for the bidder drop control. Thus, in this case, the size of the DL class decreases (i.e.,  $N_{dl}$  decreases). The total revenue of the service provider, at least initially, increases with the reservation price (this increase is the largest when  $N_{dw} = R$  and then it steadily shrinks and may become negative for larger reservation prices, when  $N_{dw}$  is smaller). However, increasing the number of the network resource units reserved for the bidder drop control decreases fairness of the network resource allocation. This is because the winners in the first phase of the



VLLF algorithm are selected based not only on their current bidding prices but also on their bidding prices and status in the previous auction round.

The reverse case (i.e., decreasing the reservation price) increases fairness by decreasing the number of units available for the bidder drop control, and may either increase or decrease the total revenue of the service provider. Accordingly, in deciding the upper bound of the reservation price, the network service provider should balance an increase in the total revenues versus the loss of fairness induced by the selected reservation price.

Reservation Price	$N_{dw}$	WPWL	$N_{dl}$	Revenue	Fairness
↑	↓	↑	↓	↑ for $N_{dw} \sim R$	↓
↓	↑	↓	↑	↓ for $N_{dw} \sim R$	↑

↑ : Increase ↓ : Decrease ~: Close to

**Table 1:** The interrelationship between the customer's classes

Based on many experiments conducted under the various customer wealth distributions, described in Appendix B in [17], we discerned **2/3 rule** (i.e., every two out of three network resources should be allocated to the DW class) for the near optimal distribution of the network resources between the DW class and the pool of network resources reserved for the VLLF bidder drop control algorithm. This rule leads to a simple and adaptive formula for the network service provider optimal bidding price  $b_0$  as equal to the  $2R/3$  highest bid in the current auction round.

### 3.4.3 TW class average bidding price approach

In analysis of our experimental results (see APPENDIX C in [17]), we observed the following: In most cases, the average bidding price of bidders in the TW class, if used as the reservation price of network service provider, results in near optimal revenue for the service provider. Based on these experimental results, we propose a simple and adaptive way for establishing the network service provider's reservation price.

In each auction round, after the network service provider ranks customer's bidding prices in ascending order, the average  $P_{atw}$  of the bidding prices of bidders in the TW class is computed. Following inequality (8), the network service provider selects larger of  $P_{atw}$  and  $C_m$  as her bidding price  $b_0$  in the current auction round.

## 4. Analysis of experimental results

### 4.1 Experimentation scenarios

#### 4.1.1 Auction mechanism

In our experiments, we compare the following three auction mechanisms for the single item, multiple winners, discriminatory pricing, and sealed bid recurring auction for the short-term contracts on homogeneous network services:

- 1) **Traditional Auction (TA):** an auction mechanism with no bidder drop control mechanism, so bidders drop during the recurring auction as a result of starvation.
- 2) **Traditional Auction with No Bidder Drop Assumption (TANBDA):** an auction mechanism in which bidders never drop during the recurring auction in spite of starvation (i.e., despite the consecutive losses in the recurring auction).
- 3) **Auction with BDC Winner Policy (BDC):** an auction mechanism with the VLLF bidder drop control algorithm and the TW class average bidding approach described in the previous sections.

#### 4.1.2 Wealth distribution of customers and the minimum cost of a unit network resource

In addition to the perceived intrinsic value of the traded goods, the wealth of each customer limits her willingness to pay and defines her true valuation of the traded goods. For simplicity, we consider only the distribution of the customer's true valuation here. We set the network service provider's minimum cost of a unit of the network resources at  $C_m = 5$ . We consider also three distributions, all with mean of 5, of the customers true valuations: (1) the exponential distribution, (2) the uniform distribution over the range  $[0, 10]$ , and (3) the Gaussian distribution.

#### 4.1.3 Bidders, bidding behavior and goods

There are 100 customers (i.e., bidders) in our experiments. We assume that the initial bidding price is randomly selected from the range  $[t_i / 2, t_i]$ , where  $t_i$  represents the true valuation of the traded goods by customer  $i$ . Following the sealed bidding assumption, bidders bid independently of each other. However, in a recurring auction, the bidding behavior is influenced by results of the previous auction rounds based on the win/loss outcome informed to each bidder. According to the assumption of risk neutral bidders, the bidders will maximize the expected profit. Hence, we assumed the following bidding behavior.

If a bidder lost in the last auction round, he increments his bidding price by  $\alpha \cdot b_i^l$  to increase his win probability in the current auction round.  $0 \leq \alpha \leq 1$  denotes the increase coefficient and  $b_i^l$  represents the bidding price of bidder  $i$  in the last auction round. The change in the bidding price is limited by the bidder's true valuation. If the bidder won in the last auction round, she, with probability of 0.5, either decreases the bidding price by a factor  $\beta$  or maintains it unchanged. The decrease attempts to maximize the expected profit.  $\alpha$  and  $\beta$  are set in the experiments to 0.2. The minimum bidding price of a bidder is 0.1. If a customer drops out of an auction, his bidding price is set to 0. This means that the dropped bidders don't ever return to the auction.

There are 50 units of resources available for allocation in each auction round.

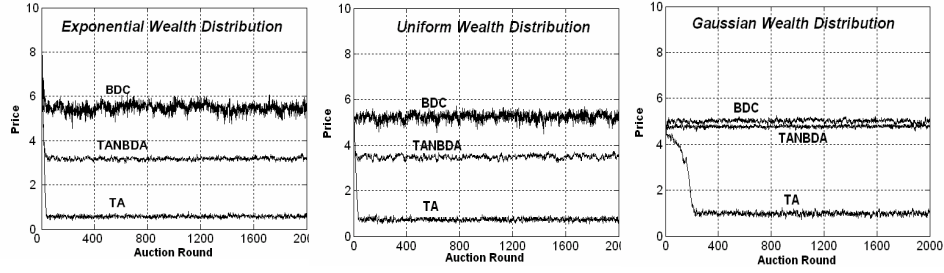
#### 4.1.4 Tolerance of Consecutive Losses (TCL)

The customer's tolerance of consecutive losses, abbreviated as TCL, denotes the maximum number of consecutive losses that a customer can tolerate before dropping out of an auction. TCL of each customer is uniformly distributed over the range of  $[2, 10]$ .

Based on the above experimental scenarios, we assume that, initially, each bidder wants to participate in an auction round and the auction is executed 2000 times.

## 4.2 Discussion of experimental results

Our experiments focus on the network service provider's revenue and the network resource allocation fairness. An auction is entirely fair if a bidder with a bid higher than any of the winners is a winner as well. In our experiments, the network service provider's revenue is proportional to the average bidding price of winners in each auction round, so we use the latter as a measure of the former. We also measure the number of wins for each customer in 2000 rounds of the recurring auction. The resulting distribution is a metric of fairness, because higher bidding customers should be more frequent winners than the lower bidding ones.



**Figure 5:** The average bidding price of winners

The experimental results of TANBDA are impossible to achieve in the real recurring auction, because the no bidder drop assumption is unrealistic. In the real world, starvation will be triggered by the uneven distribution of customer's true valuations. The starvation combined with the customer's ability to substitute the premium quality network services with the best-effort ones will motivate starving customers to drop out of auction. Thus, in our experimentation, TANBDA is used only for comparison.

Fairness of TANBDA is optimal, because a bidder with the bid higher than any of the winners is also a winner. Additionally, by the no bidder drop assumption, TANBDA never loses a customer with high true valuation but low TCL. This means that TANBDA prevents the loss of fairness that may result from low TCL. Thus, we can measure the loss of fairness of TA and BDC mechanisms by their degree of deviation from the fairness of TANBDA. We measure the auction fairness  $LF_k$  for auction mechanism  $k$  by the distribution of wins between the customers:

$$LF_k = \frac{\sum_{i=1}^n |NW_{TANBDA}(i) - NW_k(i)|}{R \cdot N_{Total\_Auction}} \cdot 100, \quad (9)$$

where  $n$  denotes the total number of customers in the recurring auction,  $NW_{TANBDA}(i)$  and  $NW_k(i)$  represent the total number of wins by bidder  $i$  during  $N_{Total\_Auction}$  of auction rounds in TANBDA and auction mechanism  $k$ , respectively.

As shown in Figure 5 under various wealth distributions, TA cannot maintain the network service provider's desired revenue in a recurring auction. The inevitable

bidders' drops decrease the price competition between customers who remaining in the auction. Accordingly, the remaining customers decrease their bidding price in the forthcoming auction rounds to maximize their expected profit. In the long run, the revenue of each auction round plunges to a very low level (i.e., below 1.0), compared to the network service provider minimum cost (here 5.0). An efficient bidder drop control, however, can maintain the number of bidders permanently high in each auction round. Thus, the network service provider can preserve the near optimal level of the desired revenue in each auction round.

Remarkably, as shown in Table 2, the loss of fairness of BDC is lower than that of TA under the various wealth distributions, when bidders are allowed to drop out of an auction. The reason is that TA cannot prevent the loss of fairness caused by the higher bidders dropping out of an auction as a result of exceeding their TCL (i.e., TA cannot prevent a customer who is willing to pay high prices but has the low TCL from dropping out of an auction). Even though TA maintains higher fairness than BDC in each auction round, because customers in the TW class become winners in TA, remarkably, BDC suffers lower loss of fairness during the recurring auction. Apparently, loss of fairness that results from low TCL is the dominating factor in the recurring auction.

	Exponential	Uniform	Gaussian
TA	34.6 %	23.9 %	29.4 %
BDC	9.4 %	6.0 %	11.9 %

**Table 2:** Loss of fairness

We also simulated the more general case of an auction in which a bidder who dropped out can return when the winning price becomes sufficiently low. For this case, the experimental results show that the revenue of the network service provider settles somewhere between the revenues of TA and TANBDA. This is not surprising because TA and TANBDA are the border cases of the general model. The revenue of the TA case sets the lower bound of the revenue of the general case because there are no bidders returning during the recurring auction. The revenue of TANBDA sets the upper bound because all bidders return immediately to the recurring auction in that case.

In conclusion, BDC achieves the increased revenue and decreased loss of fairness in a recurring auction compared to the traditional auction mechanisms. Additionally, the winning distribution of the BDC auction shows that the proposed pricing mechanism can achieve broader winning distribution than TA. Even poor customers have a non-zero probability of winning in BDC auction thanks to the VLLF bidder drop control algorithm. For this reason, wins are widely distributed over a wide range of the customer wealth. This property of our mechanism is important when the network resources are viewed as a public resource.

In short, the proposed BDC auction stabilizes the network service provider's revenue, minimizes the loss of fairness, and broadens winning distribution under the trade-off relationship between these three goals.

## 5. Conclusions and future works

During a recurring auction, customers can drop out of the auction at any time. For this reason, keeping customers interested in participating in the auction stabilizes the market by preventing a collapse of the price competition. Hence, the bidder drop problem is one of the most important factors in designing winner selection algorithms in the recurring auction for network services. The resource waste is another problem that needs to be considered when maximizing the revenues from the allocation of time sensitive network resources.

We have presented a novel auction mechanism (i.e., an auction with the BDC winner policy) for a recurring auction for short-term contracts on network services. This mechanism stabilizes the market by preventing a price collapse thanks to the efficient bidder drop control. Compared to the traditional auction mechanisms, the proposed mechanism increases the network service provider's revenue by maintaining the price competition and at the same time it decreases loss of fairness of the network resource allocation by preventing the bidders who are willing to bid high prices from dropping out of the auction. Additionally, the pattern of the winning price distribution shows that an auction with the BDC winner policy, when compared to the traditional auction, supports larger active customer base by occasionally allocating resources to the customers with lower resource valuation.

In our future work, we intend to design more efficient bidder drop control algorithms and to provide a stronger theoretical foundation of the proposed auction mechanism. We will also consider a more general case of an auction in which multiple heterogeneous network services are traded in a recurring auction.

### APPENDIX A: Proof of optimality of the bidding price

#### 1) Multiple winners, discriminatory pricing, sealed bid auction with reservation price:

Based on the assumptions made in this paper, each customer (i.e., bidder)'s expected profit  $U_i$  in each auction round is

$$U_i = (t_i - b_i) \cdot \text{Pr}_{win}, \quad (\text{A.1})$$

where  $t_i$  denotes the private true valuation of bidder  $i$ ,  $b_i$  represents the bidding price of bidder  $i$ , and  $\text{Pr}_{win}$  denotes the probability that the bidder wins with the bidding price  $b_i$  in the current auction round. By using Equation (A.1) and the assumption that each bidder randomly selects the bidding price from the range  $[0, t_i]$  with the uniform probability, the expected profit of bidder  $i$  is

$$U_i = (t_i - b_i) \cdot \left[ \left( \frac{t_i - b_0}{t_i} \right) \cdot \prod_{j=1}^{n-k} \frac{b_i}{t_j} \right] \quad (\text{A.2})$$

As bidders are risk neutral,  $b_i$  that maximizes  $U_i$  can be found at zero of  $U_i$  derivative that yields the following optimal bidding price in our negotiation scenarios:

$$b_i^* = \left( \frac{n-k}{n-k+1} \right) \cdot t_i \quad (\text{A.3})$$

#### 2) Multiple winners, discriminatory pricing, sealed bid auction without reservation price:

Each bidder's expected profit is defined by

$$U_i = (t_i - b_i) \cdot \prod_{j=1}^{n-k} \frac{b_j}{t_j} \quad (\text{A.4})$$

By using Equation (A.4) and the same reasoning as in the reservation price case, we get the same result as in Equation (A.3).

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