Possible Winners When New Alternatives Join: New Results Coming Up!

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Abstract

In a voting system, sometimes multiple new alternatives will join the election after the voters' preferences over the initial alternatives have been revealed. Computing whether a given alternative can be a co-winner when multiple new alternatives join the election is called the *possible co-winner with new alternatives (PcWNA)* problem, introduced by Chevaleyre et al. [4, 5]. In this paper, we show that the PcWNA problems are NP-complete for the Bucklin, Copeland$_0$, and Simpson (a.k.a. maximin) rule, even when the number of new alternatives is no more than a constant. We also show that the PcWNA problem can be solved in polynomial time for plurality with runoff. For the approval rule, we define three different ways to extend a linear order with new alternatives, and characterize the computational complexity of the PcWNA problem for each of them.

1 Introduction

In many real-life situations, a set of voters have to choose a common alternative out of a set that can grow during the process. For instance, when looking for a meeting date, it may happen that of new dates become possible. A recent paper by Chevaleyre et al. [4] considers the following problem: *suppose that the voters' preferences about a set of initial alternatives have already been elicited, and we know that a given number k of new alternatives will join the election; we ask who among the initial alternatives can possibly win the election in the end*. This problem is a special case of the *possible winner problem* [7, 9, 8, 2, 3, 1], restricted to the case where the incomplete profile consists of a collection of full rankings over the initial alternatives (nothing being known about the voters' preferences about the new alternatives), somehow dual of another special case of the problem where the incomplete profile consists of a collection of full rankings over all alternatives for a subset of voters (nothing being known about the remaining voters' preferences), which itself is equivalent to the coalitional manipulation problem.

Chevaleyre et al. [4, 5] investigated the complexity of computing possible winners with new alternatives, and laid the focus on scoring rules, obtaining both polynomiality and NP-completeness results, depending on the scoring rule used and the number of new alternatives. Their results, however, did not go beyond scoring rules. Here we go further and give results for several other common rules, especially some common rules that are based on *pairwise elections*. After giving some background in Section 2, each of the following sections is devoted to the PcWNA problem for a specific voting rule. In Section 3, we focus on approval voting. Since the notion of a complete profile (including the new alternatives) extending a partial profile over the initial alternatives is not straightforward, we propose three possible definitions, which we think are the three most reasonable definitions. We show that PcWNA problems are trivial for two of these definitions, and NP-complete for the third one. In Sections 4, 5 and 6 we show that the problem is NP-complete for, respectively, the Bucklin rule, the Copeland rule, and the Simpson (a.k.a. maximin) rule, and finally in Section 7 we focus on plurality with runoff, for which the problem is in P (due to the space constraint, the proof of this result is omitted).

2 Preliminaries

Let $\mathcal{C}$ be the set of *alternatives* (or candidates), with $|\mathcal{C}| = m$. Let $\mathcal{I}(\mathcal{C})$ denote the set of votes. Most often, the set of votes is the set of all linear orders over $\mathcal{C}$. An *n-profile* P is a collection of n votes for some $n \in \mathbb{N}$, that is, $P \in \mathcal{I}(\mathcal{C})^n$. A *voting rule* r is a mapping that assigns to each profile a set of winning alternatives, that is, r is a mapping from $\{\emptyset\} \cup \mathcal{I}(\mathcal{C}) \cup \mathcal{I}(\mathcal{C})^2 \cup \ldots$ to $2^{\mathcal{C}}$. Some common voting rules are listed below. For all of them (except the approval rule), $\mathcal{I}(\mathcal{C})$ is the set of all linear orders over $\mathcal{C}$; for the approval rule, the set of votes is the set of all subsets of $\mathcal{C}$, that is, $\mathcal{I}(\mathcal{C}) = \{S : S \subseteq \mathcal{C}\}$.
(Positional) scoring rules: Given a *scoring vector* $\vec{v} = (v(1), \ldots, v(m))$, for any vote $V \in L(\mathcal{C})$ and any $c \in \mathcal{C}$, let $s(V, c) = v(j)$, where j is the rank of c in V. For any profile $P = (V_1, \ldots, V_n)$, let $s(P, c) = \sum_{i=1}^{n} s(V_i, c)$. The rule will select $c \in \mathcal{C}$ so that $s(P, c)$ is maximized. Some examples of positional scoring rules are *Borda*, for which the scoring vector is $(m-1, m-2, \ldots, 0)$; *l-approval* ($l \leq m$), for which the scoring vector is $v(1) = \ldots = v(l) = 1$ and $v_{l+1} = \ldots = v_m = 0$; and *plurality*, for which the scoring vector is $(1, 0, \ldots, 0)$.
Approval: Each voter submits a set of alternatives (that is, the alternatives that are "approved" by the voter). The winner is the alternative approved by the largest number of voters. Note that the approval rule is different from the l-approval rule, in that for the l-approval rule, a voter must approve l alternatives, whereas for the approval rule, a voter can approve an arbitrary number of alternatives.
Bucklin: The Bucklin score of an alternative c is the smallest number t such that more than half of the votes rank c among top t positions. The alternatives that have the lowest Bucklin score win. (We do not consider any further tie-breaking for Bucklin.)
Copeland$_\alpha$ ($0 \leq \alpha \leq 1$): For any two alternatives c_i and c_j, we can simulate a *pairwise election* between them, by seeing how many votes prefer c_i to c_j, and how many prefer c_j to c_i; the winner of the pairwise election is the one preferred more often. Then, an alternative receives one point for each win in a pairwise election, α points for each tie, and zero point for each loss. The alternatives that have the highest scores win.
Simpson (a.k.a. maximin): Let $N_P(c_i, c_j)$ denote the number of votes that rank c_i ahead of c_j in P. The Simpson score of alternative $c \in C$ in profile P is defined as $Sim_P(c) = \min\{N_P(c, c') : c' \in C \setminus \{c\}\}$. A Simpson winner for P is an alternative $c_0 \in C$ such that $Sim_P(c_0) = \max\{Sim_P(c) : c \in C\}$.
Plurality with runoff: The election has two rounds. In the first round, all alternatives are eliminated except the two with the highest plurality scores. In the second round (runoff), the winner is the alternative that wins the pairwise election between them.

Let $\mathcal{C}$ denote the set of original alternatives, let Y denote the set of new alternatives. For any linear order V over $\mathcal{C}$, a linear order V' over $\mathcal{C} \cup \{V\}$ *extend* V, if in V', the pairwise comparison between any pair of alternatives in $\mathcal{C}$ is the same as in V. That is, for any $c, d \in \mathcal{C}$, $c \succ_V d$ if and only if $c \succ_{V'} d$.

Given a voting rule r, an alternative c, and a profile P over $\mathcal{C}$, we are asked whether there exists a profile P' over $\mathcal{C} \cup Y$ such that P' is an extension of P and $c \in r(P')$. This problem is called the *possible co-winner with new alternatives (PcWNA)* problem [4, 5].

Similarly, we let *PWNA* denote the problem in which we are asked whether c is a possible (unique) winner, that is, $r(P') = \{c\}$. Up to now, the PcWNA and PWNA problems are well-defined for all voting rules studied in this paper (except the approval rule). For the approval rule, we will introduce three types of extension, and discuss the computational complexity of the PcWNA and PWNA problems under these extensions.

In this paper, all NP-hardness results are proved by reductions from the Exact Cover by 3-Sets problem (denoted by X3C) or the 3-DIMENSIONAL MATCHING problem (denoted by 3DM). An instance $I = (\mathcal{S}, \mathcal{V})$ of X3C consists of a set $\mathcal{V} = \{v_1, \ldots, v_{3q}\}$ of $3q$ elements and $t \geq q$ 3-sets $\mathcal{S} = \{S_1, \ldots, S_t\}$ of $\mathcal{V}$, i.e., for any $i \leq t$, $S_i \subseteq \mathcal{V}$ and $|S_i| = 3$. For any $v \in \mathcal{V}$, let $d_I(v)$

denote the number of 3-sets containing element v in instance I. Let $\Delta(I) = \max_{v \in \mathcal{V}} d_I(v)$. We are asked whether there exists a subset $J \subseteq \{1, \dots, t\}$ such that $|J| = q$ and $\bigcup_{j \in J} S_j = \mathcal{V}$ (indeed, the sets S_j for $j \in J$ form a partition of $\mathcal{V}$). This problem is known to be NP-complete, even if $\Delta(I) \leq 3$ (problem [SP2] page 221 in [6]). In this paper, we will use a special case of 3DM that is also a special case of X3C, defined as follows.[1] Given A, B, X, where $A = \{a_1, \dots, a_q\}$, $B = \{b_1, \dots, b_q\}$, $X = \{x_1, \dots, x_q\}$, $T \subseteq A \times B \times X$, $T = \{S_1, \dots, S_t\}$ with $t \geq q$. We are asked whether there exists $M \subseteq T$ such that $|M| = q$ and for any $(a_1, b_1, x_1), (a_2, b_2, x_2) \in M$, we have $a_1 \neq a_2$, $b_1 \neq b_2$, and $x_1 \neq x_2$. That is, M corresponds to an exact cover of $\mathcal{V} = A \cup B \cup X$. This problem with the restriction where no element of $A \cup B \cup X$ occurs in more than 3 triples (i.e, $\Delta(I) \leq 3$) is known to be NP-complete (problem [SP1] page 221 in [6]).

It is straightforward to check that the PcWNA (respectively, PWNA) problems for all voting rules studied in this paper are in NP, because given an extension of a profile P, it is polynomial to verify if the given alternative c is a co-winner (respectively, unique winner) for all rules studied in this paper (again, we discuss the approval rule separately). Therefore, in this paper we only show NP-hardness proofs.

To prove that the PcWNA and PWNA problems are NP-hard, we first prove that another useful special case of 3DM (as well as X3C) remains NP-complete.

Proposition 1 3DM *is* NP*-complete, even if q is even, $t = 3q/2$, and $\Delta(I) \leq 6$.*

Proof of Proposition 1: Let $I = (T, A \times B \times X)$ be an instance of 3DM with $A = \{a_1, \dots, a_q\}$, $B = \{b_1, \dots, b_q\}$, $X = \{x_1, \dots, x_q\}$, $T \subseteq A \times B \times X$, $T = \{S_1, \dots, S_t\}$ and $\Delta(I) \leq 3$. We next show how to build an instance $I' = (T', A' \times B' \times X')$ of 3DM in polynomial time, with $|A'| = |B'| = |X'| = q'$, $T' \subseteq A' \times B' \times X'$ and $|T'| = t'$ such that q' is even, $t' = 3q'/2$, and $\Delta(I') \leq 6$.

• If q is odd, then we add to the instance 3 new elements $\{a'_1, b'_1, x'_1\}$ with $A' = A \cup \{a'_1\}$, $B' = B \cup \{b'_1\}$, $X' = X \cup \{x'_1\}$ and one new triples (a'_1, b'_1, x'_1).

• Suppose that q is even. If $t > 3q/2$, then we add $6(t - 3q/2)$ new elements $\{a'_1, \dots, a'_{2(t-3q/2)}\}$ to A, $\{b'_1, \dots, b'_{2(t-3q/2)}\}$ to B, $\{x'_1, \dots, x'_{2(t-3q/2)}\}$ to X and $2(t - 3q/2)$ new triples $\{S'_1, \dots, S'_{2(t-3q/2)}\}$, where for any $i \leq 2(t - 3q/2)$, $S'_i = (a'_i, b'_i, x'_i)$. If $t < 3q/2$, then we add $3q/2 - t$ dummy triples to T by duplicating $3q/2 - t$ triples of T once each. We note that $t \geq q$ implies that $t \geq 3q/2 - t$.

It is easy to check in I', q' is even, $t' = 3q'/2$, and $\Delta(I') \leq 6$. The size of the input of the new instance is polynomial in the size of the input of the old instance. Moreover, I is a yes-instance if and only if I' is also a yes-instance. □

3 Approval

Since the input of the approval rule is different from the input of other voting rules studied in this paper, we have to define the set of possible extensions of an approval profile over $\mathcal{C}$. Let $P_{\mathcal{C}} = (V_1, \dots, V_n)$ be an approval profile over $\mathcal{C}$, where each V_i is a subset of $\mathcal{C}$. An extension of $P_{\mathcal{C}}$ over $\mathcal{C} \cup Y$ is a collection $(V'_1, \dots, V'_n)$ where $V'_i \subseteq \mathcal{C} \cup Y$ is an extension of V_i. Now, we have to define what it means to say that $V' \subseteq \mathcal{C} \cup Y$ is an extension of $V \subseteq \mathcal{C}$. We can think of three natural definitions:

Definition 1 (extension of an approval vote, definition 1) *$V' \subseteq \mathcal{C} \cup Y$ is an extension of $V \subseteq \mathcal{C}$ if $V \subseteq V'$ and $V' \cap \mathcal{C} = V \cap \mathcal{C}$.*

In other words, under this definition, V' is an extension of V if $V' = V \cup Y'$, where $Y' \subseteq Y$. The problem with Definition 1 is that it assumes that any alternative approved in V is still approved in V'.

[1] Generally, 3DM is not a special case of X3C.

However, in some contexts, extending the choice with alternatives of Y may change the "approval threshold". Moreover, since we have more alternatives, this threshold should either stay the same or move upwards: some alternatives that were approved initially may become disapproved. This leads to the following definition of extension:

Definition 2 (extension of an approval vote, definition 2) *$V' \subseteq \mathcal{C} \cup Y$ is an extension of $V \subseteq X$ if one of the following conditions holds: (1) $V = V'$; (2) $V' \cap Y \neq \emptyset$ and $V' \cap \mathcal{C} \subseteq V$.*

Lastly, we may also allow the acceptance threshold to move downwards, even though the set of alternatives grows, especially in the case where the new alternatives are particularly bad, thus rendering some alternatives in $\mathcal{C}$ acceptable after all. This leads to the third definition of extension:

Definition 3 (extension of an approval vote, definition 3) *$V' \subseteq \mathcal{C} \cup Y$ is an extension of $V \subseteq \mathcal{C}$ if one of the following conditions holds: (1) $V' \cap \mathcal{C} \subset V$ and $V' \cap Y \neq \emptyset$; (2) $V \subset V' \cap \mathcal{C}$, and $Y \setminus V' \neq \emptyset$; (3) $V' \cap \mathcal{C} = V$.*

Under Definition 3, either the threshold moves upward, in which case all alternatives which were disapproved in$V$ are still disapproved in V', and obviously, at least one alternative in Y is approved; or the threshold moves downward, in which case all alternatives that were approved in V are still approved in V', and obviously not all alternatives in Y are approved. Note that in the case where $V' \cap \mathcal{C} = V$, the threshold can have moved upward, or downward, or remained the same.

Let us give a brief summary of the three definitions of extension. Definition 1 assumes that the threshold cannot move; Definition 2 assumes that the threshold can stay the same or move upward (because the set of alternatives grows); and Definition 3 assumes that the threshold can stay the same, move upward, or move downward. Next, we show an example that illustrates these definitions. Let $\mathcal{C} = \{a, b, c, d\}$, $Y = \{y_1, y_2\}$, and $V = \{a, b\}$.

- $V'_1 = \{a, b\}$ and $V'_2 = \{a, b, y_1\}$ are extensions of V under any definition;
- $V' = \{a, y_1\}$ is an extension of V under definitions 2 and 3 but not under definition 1 (the threshold has moved upward, since b was approved in V and is no longer approved in V');
- $V' = \{a, b, c, y_1\}$ is an extension of V under definition 3 but neither under definitions 1 nor 2 (the threshold has moved downward, since c was not approved in V and becomes approved in V' – note that, intuitively, y_2 must be a very unfavored alternative for this to happen);
- $V' = \{a, b, c\}$ is an extension of V under definitions 3 but neither under definitions 1 nor 2, for the same reason as above;
- $V' = \{a\}$ is not an extension of V under any of the definitions: to have b disapproved in V' and approved in V, the threshold has to move upward, which cannot be the case if no alternative of Y is approved;
- $V' = \{a, b, c, y_1, y_2\}$ is not an extension of V under any of the definitions: to have c disapproved in V and approved in V', the threshold has to move downward, which cannot be the case of all alternatives of Y are disapproved;
- $V' = \{a, c, y_1\}$ is not an extension of V under any of the definitions: the threshold cannot simultaneously move upward and downward.

It is straightforward to check that the PcWNA and PWNA problems are in P for approval under definition 1: an alternative $c \in \mathcal{C}$ is a possible (co-)winner in P if and only if it is a (co-)winner for approval in P (this is because for any $V \in P$, the scores of alternatives in $\mathcal{C}$ will not change from V to its extension V'). However, when we adopt definition 2 of extension, the problems become NP-complete.

Theorem 1 *Under Definition 2, PcWNA and PWNA problems are NP-complete for the approval rule.*

Proof of Theorem 1: We first prove the hardness of the PcWNA problem by a reduction from X3C. For any X3C instance $I = (\mathcal{S}, \mathcal{V})$, we construct the following PcWNA instance.

Alternatives: $\mathcal{V} \cup \{c\} \cup Y$, where $Y = \{y_1, \dots, y_{t-q}\}$.

Votes: for any $i \le t$, we have a vote $V_i = S_i$; and we have an additional vote $V_{t+1} = \{c\}$. That is, $P = (V_1, \dots, V_t, V_{t+1})$.

Suppose the X3C instance has a solution, denoted by $\{S_{i_1}, \dots, S_{i_q}\}$. Then, take the following extension P' of P: for any $j \le q$, let $V'_{i_j} = V_{i_j}$. For any $i \le t$ such that $i \ne i_j$ for any $j \le q$, we let V'_i be a singleton containing exactly one of the new alternatives. Let $V'_{p+1} = \{c\}$. For any $v \in \mathcal{V}$, because v appears exactly in one S_{i_j}, v is approved by exactly one voter. So is c. Now, there are exactly $t - q$ votes V_i where i is not equal to one of the i_j's. Therefore, the total approval score of the new alternatives is $t - q$, and it suffices to approve every new alternative exactly once. Therefore c is a co-winner in P', and thus a possible co-winner in P.

Conversely, suppose c is a possible co-winner for P and let P' be an extension of P for which c is a co-winner. We note that c is approved at most once in P'. Therefore, every alternative in $\mathcal{V} \cup Y$ must be approved at most once. Without loss of generality, assume that every vote V'_i in P' is either of the form V_i or of the form $\{y_j\}$ (if not, remove every alternative (except one y_j) from V'_i; c will still be a co-winner in the resulting profile). Since we have $t - q$ new alternatives, each being approved at most once in P', we have at least q votes V'_i in P' such that $V'_i = V_i$. If we had more than q votes V'_i such that $V'_i = V_i$, then more than $3q$ points would be distributed to $3q$ alternatives and one of them would get at least 2, which means that c would not be a co-winner in P'. Therefore we have exactly q votes V'_i such that $V'_i = V_i$, and $3q$ points distributed to $3q$ alternatives; since none of them gets more than one point, they get one point each, which implies that the collection of all S_i such that $V_i = V'_i$ forms an exact cover of C.

For the PWNA problem, we add one more vote $V_{t+2} = \{c\}$ to the profile P. □

Now, let us consider Definition 3. Notice that the profile P' where every voter adds c to her vote (if she was not already voting for c) is an extension of P, and obviously c is a co-winner in P', therefore every alternative of $\mathcal{C}$ is a possible co-winner for P, which means that the problem is trivial.

4 Bucklin

Theorem 2 *The PWNA and PcWNA problems are* NP*-complete for Bucklin, even when there are three new alternatives.*

Proof of Theorem 2: We prove the NP-hardness of the PcWNA problem by a reduction from the special case of 3DM mentioned in Proposition 1. Given any 3DM instance where $|A| = |B| = |X| = q$, q is even, $t = 3q/2$, and no element in $A \cup B \cup X$ appears in more than 6 elements in T, we construct a PcWNA instance as follows. Without loss of generality, assume $q \ge 5$; otherwise the instance 3DM can be solved in linear time.

Alternatives: $A \cup B \cup X \cup Y \cup D \cup \{c\}$, where $Y = \{y_1, y_2, y_3\}$ is the set of new alternatives, and $D = \{d_1, \dots, d_{9q^2}\}$ is the set of auxiliary alternatives.

Votes: For any $i \le 2q$, we define a vote V_i. Let $P = (V_1, \dots, V_{2q+1})$. Instead of defining these votes explicitly, below we give the properties that P satisfies. The votes can be constructed in polynomial time.

(i) For any $i \le q$, c is ranked in the first position. Suppose $S_i = (a, b, x)$. Then, let a, b, x be ranked in the $(3q + 1)$th, $(3q + 2)$th, and $(3q + 3)$th positions in V_i, respectively.

(ii) For any i such that $q < i \le 3q/2 = t$, c is ranked in the $(3q + 4)$th position. Suppose $S_i = (a, b, x)$. Then, let a, b, x be ranked in the $(3q + 1)$th ,$(3q + 2)$th, and $(3q + 3)$th positions in V_i, respectively.

(iii) For any i such that $3q/2 < i \le 2q + 1$, let c be ranked in the $(3q + 4)$th position, and no alternative in $A \cup B \cup X$ is ranked in the $(3q + 1)$th, $(3q + 2)$th, or $(3q + 3)$th position in V_i.

(iv) For any $j \leq 3q$, v_j is ranked within top $3q+3$ positions for exactly $q+1$ times in P.

(v) For any $d \in D$, d is ranked within top $3q+4$ positions at most once.

The existence of a profile P that satisfies (iv) is guaranteed by the assumption that in the 3DM instance, $q \geq 5$, no element is covered by more than 6 times, and there are enough positions within top $3q+3$ positions in all votes to fit in all alternatives in $\mathcal{C}$, with each alternative appears $q+1$ times. We note that there are in total $9q^2$ auxiliary alternatives, and the total number of top $3q+4$ positions in all votes is $(3q+4)(2q+1) < 9q^2$. Therefore, (v) can be satisfied. It follows that there exists a profile P that satisfies (i), (ii), (iii), (iv), and (v), and such a profile can be constructed in polynomial time (by first putting the alternatives to their positions defined in (i), (ii), and (iii), then filling out the positions using remaining alternatives to meet conditions (iv) and (v)). The Bucklin score of c is $3q+4$ in P. For any $j \leq q$, the Bucklin score of a_j (resp., b_j, x_j) is at most $3q+3$ in P, and for any $j \leq 9q^2$, the Bucklin score of $d_j \in D$ is at least $3q+4$ in P. Observe that the Bucklin score of any alternative cannot increase in any extension of P.

Suppose that the 3DM instance has a solution, denoted by $\{S_j : j \in J\}$, where $J \subseteq \{1,\ldots,t\}$. For any $j \in J$, we let V'_j be the extension of V_j in which y_1, y_2, y_3 are ranked in the $(3q+1)$th, $(3q+2)$th, and $(3q+3)$th positions, respectively. For any $j \in \{1,\ldots,2q+1\} \setminus J$, we let V'_j be the extension of V_j where $\{y_1, y_2, y_3\}$ are ranked in the bottom positions. Let $P' = (V'_1,\ldots,V'_{2q+1})$. It follows that in P', the Bucklin score of c is $3q+4$, and the Bucklin score of any other alternative is at least $3q+4$. Therefore, c is a co-winner for Bucklin for P', which means that there is a solution to the PcWNA instance.

Conversely, suppose that there is a solution to the PcWNA instance, denoted by $P' = (V'_1,\ldots,V'_{2q+1})$. We recall that in order for c to be a co-winner, the Bucklin score of any alternative in $A \cup B \cup X$ must be at least $3q+4$ (since the Bucklin score of c cannot increase in P'). We note that there are only three new alternatives, and the $(3q+1)$th, $(3q+2)$th, and $(3q+3)$th positions in V_i are occupied by some alternatives in D. It follows that for every $a \in A$ and every i such that $t < i \leq 2q+1$, it cannot be the case that a is ranked within top $3q+3$ positions in V_i, and a is ranked lower than the $(3q+3)$th position in V'_i. Therefore, for every $a \in A$, there exists $i \leq t$ such that a is ranked within top $3q+3$ positions in V_i, and is ranked lower than the $(3q+3)$th position in V'_i. It follows that in each of such V'_i where a is ranked lower than the $(3q+3)$th position, the new alternatives must be ranked within top $3q+3$ positions. Therefore, each new alternative must be ranked within top $3q+3$ positions in $V_1,\ldots,V_t$ for q times (one for each $a \in A$). Because c is a co-winner, no alternative in Y is ranked within top $3q+3$ positions in P' for more than q times. Therefore, in exactly q votes in P', the alternatives in Y are ranked within top $3q+3$ positions. We let $\{V'_{i_1},\ldots,V'_{i_q}\}$ denote these votes.

We claim that $\{S_{i_1},\ldots,S_{i_q}\}$ is a solution to the 3DM instance. If not, then there exists $e \in B \cup X$ that does not appear in any S_{i_j}. However, it follows that e is ranked within top $3q+3$ positions for exactly q times, which means that the Bucklin score of e is at most $3q+3$. This contradicts the assumption that c is a co-winner for P'. Therefore, the PcWNA problem is NP-hard for Bucklin.

For PWNA, we make the following changes. In conditions (i) and (ii) that P should satisfy, we require that a, b, x are in the $(3q+2)$th, $(3q+3)$th, and $(3q+4)$th positions, respectively. □

5 Copeland$_0$

For any profile P, the Copeland score of an alternative $c \in \mathcal{C}$ in profile P is denoted by $\text{CS}_P(c) = |\{c' \in \mathcal{C} : N_P(c, c') > n/2\}|$ (recall that we focus on Copeland$_0$, which means that the tie in a pairwise election gives 0 point to both participating alternatives). We have the following straightforward observation.

Property 1 *For any profile P' over $\mathcal{C} \cup \{y\}$ that is an extension of profile P, the following inequalities hold:*

$$\forall c \in \mathcal{C},\ CS_P(c) \leq CS_{P'}(c) \leq CS_P(c) + 1 \tag{1}$$

We prove that a useful restriction of X3C remains NP-complete.

Proposition 2 X3C *is* NP*-complete, even if* $t = 2q - 2$ *and* $\Delta(I) \leq 6$.

Proof of Proposition 2: The proof is similar to the proof for Proposition 1. Let $I = (\mathcal{S}, \mathcal{V})$ be an instance of X3C, where $\mathcal{V} = \{v_1, \ldots, v_{3q}\}$ and $\mathcal{S} = \{S_1, \ldots, S_t\}$. We next show how to build an instance $I' = (\mathcal{S}', \mathcal{V}')$ of X3C in polynomial time, with $|\mathcal{V}'| = 3q'$ and $|\mathcal{S}'| \leq 6$ such that $t' = 2q' - 2$ and $\Delta(I') \leq 6$.

• If $t < 2q - 2$, then we add $2q - 2 - t$ dummy 3-sets to $\mathcal{S}$ by duplicating $2q - 2 - t$ sets of $\mathcal{S}$ once each. It follows from $t \geq q$ that $2q - 2 - t \leq q - 2 < t$.

• If $t > 2q - 2$, then we add $3(t - 2q + 2)$ new elements $v'_1, \ldots, v'_{3(t-2q+2)}$ and $t - 2q + 2$ 3-sets $\{v'_1, v'_2, v'_3\}, \ldots, \{v'_{3(t-2q+2)-2}, v'_{3(t-2q+2)-1}, v'_{3(t-2q+2)}\}$.

The size of the input of the new instance is polynomial in the size of the input of the old instance. Moreover, I is a yes-instance if and only if I' is also a yes-instance. Finally, in the new instance I', we have: $|\mathcal{V}'| = |\mathcal{V}| = 3q$ and $t' = |\mathcal{S}'| = t + (2q - 2 - t) = 2q - 2 = 2q' - 2$ in the first case, while $3q' = |X'| = 3q + 3(t - 2q + 2) = 3(t - q + 2)$ and $t' = |\mathcal{S}'| = t + (t - 2q + 2) = 2(t - q + 1) = 2(q' - 1)$ in the second case. Moreover, $d_{I'}(v) \leq 2d_I(v) \leq 6$ if $v \in \mathcal{V}$, and $d_{I'}(v) = 1$ if $v \in \mathcal{V}' \setminus \mathcal{V}$. □

Theorem 3 *The PcWNA problem is* NP*-complete for Copeland$_0$, even when there is one new alternative.*

Proof of Theorem 3: The proof is by a reduction from X3C. Let $I = (\mathcal{S}, \mathcal{V})$, where $t = 2q - 2$ and $\Delta(I) \leq 6$ be an instance of X3C as described in Proposition 2. As previously, assume $q \geq 8$; hence $\Delta(I) \leq q - 2$. For any X3C instance, we construct the following PcWNA instance for Copeland$_0$.

Alternatives: $\mathcal{V} \cup D \cup Y \cup \{c\}$, where $D = \{d_1, \ldots, d_t\}$ and $Y = \{y\}$ is the set of the new alternative.

Votes: For any $i \leq t$, we define the following $2t$ votes.

$$V_i = [d_i \succ (D \setminus \{d_i\}) \succ (\mathcal{V} \setminus S_i) \succ c \succ S_i]$$

$$V'_i = [\text{rev}(S_i) \succ \text{rev}(\mathcal{V} \setminus S_i) \succ \text{rev}(D \setminus \{d_i\}) \succ c \succ d_i]$$

Here the elements in a set are ranked according to the order of their subscripts, i.e., if $S_i = \{v_2, v_5, v_7\}$, then the elements are ranked as $v_2 \succ v_5 \succ v_7$. For any set X such that $X \subset \mathcal{V}$ or $X \subset D$, let rev(X) denote the linear order where the elements in X are ranked according to the reversed order of their subscripts. For example, rev$(\{v_2, v_5, v_7\}) = v_7 \succ v_5 \succ v_2$.

We also define the following $t = 2q - 2$ votes.

$$W_1 = \ldots = W_{q-1} = [\mathcal{V} \succ D \succ c]$$

$$W'_1 = \ldots = W'_{q-1} = [\text{rev}(D) \succ \text{rev}(\mathcal{V}) \succ c]$$

Let $P = (V_1, V'_1, \ldots, V_t, V'_t, W_1, W'_1, \ldots, W_{q-1}, W'_{q-1})$.

We note that there are $3t$ votes in the instance. We recall that by assumption, $3t/2 = 3q - 3$. We make the following observations on the function N_P.

- *For any $d \in D$, d beats c*: this holds because $N_P(c, d) = 1$.
- *For any $v \in \mathcal{V}$, v beats c*: this holds because $N_P(c, v) = d_I(v) \leq q - 2 < 3q - 3$.
- *For any $d \in D$ and $v \in \mathcal{V}$, d and v are tied*: this holds because $N_P(v, d) = t + q - 1 = 3q - 3$.

- *For any $v, v' \in \mathcal{V}$ ($v' \neq v$), v and v' are tied*: this holds because $N_P(v,v') = t+q-1 = 3q-3$, because for any $i \leq q$, $v \succ v'$ either in V_i or in V_i'.

- *For any $d, d' \in D$ ($d' \neq d$), d and d' are tied*: this holds because $N_P(d,d') = 3q-3$.

From these observations we have the following calculation on the Copeland scores:

- $\text{CS}_P(c) = 0$.
- For any $v \in \mathcal{V}$, $\text{CS}_P(v) = 1$.
- For any $d \in D$, $\text{CS}_P(d) = 1$.

Now, assume that $I = (\mathcal{S}, \mathcal{V})$ is a yes-instance of X3C; hence, there exists $J \subset \{1,\ldots,t\}$ with $|J| = q$ and $\bigcup_{j\in J} S_j = \mathcal{V}$. Next, we show how to make c a co-winner by introducing one new alternative y.

- For any $j \in J$, we let $\widetilde{V}_j = [d_j \succ D \setminus \{d_j\} \succ \mathcal{V} \setminus S_j \succ c \succ y \succ S_j]$ be the completion of V_j.
- For any $i \leq t$, we let $\widetilde{V}_i' = [\text{rev}(S_i) \succ \text{rev}(\mathcal{V} \setminus S_i) \succ \text{rev}(D \setminus \{d_i\}) \succ c \succ y \succ d_i]$ be the completion of V_i'.
- For any vote not mentioned above, we put y in the top position.
- Finally, let P' denote the profile obtained in the above way.

It follows that y loses to c in their pairwise election, and for any other alternative $c' \in \mathcal{C}$ ($c' \neq y$ and $c' \neq c$), c' and y are tied in their pairwise election. Therefore, the Copeland score is 1 for c, any alternative in $\mathcal{V}$, and any alternative in D; the Copeland score of y is 0. It follows that c is a co-winner.

Next, we show how to convert a solution to the PcWNA instance to a solution to the X3C instance. Let $P' = (\widetilde{V}_1,\ldots,\widetilde{V}_t,\widetilde{V}_1',\ldots,\widetilde{V}_t',\widetilde{W}_1,\widetilde{W}_1',\ldots,\widetilde{W}_{q-1},\widetilde{W}_{q-1}')$ be a profile with the new alternative, such that c becomes a co-winner according to the Copeland$_0$ rule. We denote $P_1' = (\widetilde{V}_1,\ldots,\widetilde{V}_t)$, $P_2' = (\widetilde{V}_1',\ldots,\widetilde{V}_t')$ and $P_3' = (\widetilde{W}_1,\widetilde{W}_1',\ldots,\widetilde{W}_{q-1},\widetilde{W}_{q-1}')$. It follows from the above observations on Copeland scores of alternatives in profile P and inequalities (1) of Property 1, that $\text{CS}_{P'}(c) = 1$, $\forall c' \in D \cup \mathcal{V}$, $\text{CS}_{P'}(c) = 1$ and $\text{CS}_{P'}(y) \leq 1$.

We now claim the following.

(a) $\forall v \in \mathcal{V}$, $N_{P'}(v,y) \leq 3q-3$, $N_{P'}(y,c) = 3q-2$ and $\forall d \in D$, $N_{P'}(d,y) = 3q-3$. $N_{P_2'}(c,y) = t = 2q-2$. Moreover, for any $i \leq q$, $c \succ y \succ d_i$ in $\widetilde{V}_i'$.

(b) $\forall v \in \mathcal{V}$, $N_{P_2'\cup P_3'}(v,y) \geq N_{P_2'\cup P_3'}(c,y)$.

For (a). Since c is a co-winner for P', c must beat y in their pairwise election. Meanwhile, any $c' \in \mathcal{V} \cup D$ cannot beat y in their pairwise elections. Therefore, we must have that $N_{P'}(c,y) \geq 3q-2$, and for any $c' \in \mathcal{V} \cup D$, $N_{P'}(c',y) \leq 3q-3$. For any $d_i \in D$, in profile P', we have that $d_i \succ c$ except in $\widetilde{V}_i'$, which means that $N_{P'}(d_i,y) \geq N_{P'}(c,y) - 1$ by transitivity in each vote. Hence, $3q-3 \geq N_{P'}(d_i,y) \geq N_{P'}(c,y) - 1 \geq 3q-3$, which means that $N_{P'}(d_i,y) = 3q-3$ and $N_{P'}(c,y) = 3q-2$. From these equalities, we deduce that $\forall d \in D$, $N_{P'}(d,y) = N_{P'}(c,y) - 1$ and then, for any $i \leq t$, we have that $c \succ y \succ d_i$ in $\widetilde{V}_i'$.

For (b). Since in P', $v \succ c$ except for some votes in P_1', we have that for all $v \in \mathcal{V}$, $N_{P_2'\cup P_3'}(v,y) \geq N_{P_2'\cup P_3'}(c,y)$.

Let $J = \{j \leq t : c \succ y \text{ in } \widetilde{V}_j\}$. We will prove that $|J| = q$ and $\cup_{j\in J} S_j = \mathcal{V}$. First, note that $|J| \leq q$ because $|J| = N_{P_1'}(c,y) \leq N_{P'}(c,y) - N_{P_2'}(c,y) = q$ from item (a).

Now, for any $v \in \mathcal{V}$ let $J_v = \{j \leq t : y \succ v \text{ in } \widetilde{V}_j\}$. We claim: $\forall v \in \mathcal{V}$, $J \cap J_v \neq \emptyset$. Otherwise, there exists $v^* \in \mathcal{V}$ with $J \cap J_{v^*} = \emptyset$. This means that $c \succ y$ implies $v^* \succ y$ in votes in P'_1. Hence, $N_{P'_1}(v^*, y) \geq N_{P'_1}(c, y)$. By adding this inequality with the inequality in item (b) (let $v = v^*$), we obtain that $N_{P'}(v^*, y) \geq N_{P'}(c, y)$. Now, combining the inequalities in item (a), we have that $3q - 3 \geq N_{P'}(v^*, y) \geq N_{P'}(c, y) = 3q - 2$, which is a contradiction. Therefore, for all $v \in \mathcal{V}$, $J \cap J_v \neq \emptyset$. Finally, since $|\mathcal{V}| = 3q$, $|S_i| = 3$ and $|J| \leq q$, we deduce that $|J| = q$ and $J = \{j \leq t : c \succ y \succ S_j \text{ in } \widetilde{V}_j\}$. Also, because for all $v \in \mathcal{V}$, $J \cap J_v \neq \emptyset$, we have $\bigcup_{j \in J} S_j = \mathcal{V}$. In conclusion, $I = (\mathcal{S}, \mathcal{V})$ is a yes-instance of X3C. This completes the NP-hardness proof for the PcWNA problem for Copeland_0. □

6 Simpson

To prove the NP-hardness of the PcWNA problem for Simpson, we first make the following observation, whose proof is straightforward.

Property 2 *Let P be a profile over $\mathcal{C}$, P' be a profile over $\mathcal{C} \cup \{y\}$, P' is an extension P. The following (in)equalities hold:*

(i) $\forall c \in \mathcal{C}$, $Sim_{P'}(c) = \min\{Sim_P(c), N_{P'}(c, y)\}$.

(ii) $\forall c \in \mathcal{C}$, $Sim_{P'}(c) \leq Sim_P(c)$.

Theorem 4 *PcWNA and PWNA problems are* NP*-complete for Simpson, even when there is one new alternative.*

Proof of Theorem 4: We first prove the NP-hardness for the PcWNA problem by a reduction from X3C. Let $I = (\mathcal{S}, \mathcal{V})$ with $t = 2q - 2$ and $\Delta(I) \leq 6$ be an instance of X3C as described in Proposition 2. Without loss of generality, assume $q \geq 8$; in particular, we deduce $\Delta(I) \leq q - 2$. We define a PcWNA instance for Simpson as follows:
Alternatives: $\mathcal{V} \cup \{c, d\} \cup \{y\}$, where y is the new alternative.
Votes: For any $i \leq t$, we define the following vote. $V_i = [(\mathcal{V} \setminus S_i) \succ d \succ c \succ S_i]$. For any $j \leq q - 1$, we define the following vote. $W_1 = \cdots = W_{q-1} = [c \succ \text{rev}(\mathcal{V}) \succ d]$. We also let $W_q = [\text{rev}(\mathcal{V}) \succ d \succ c]$. Let $P_1 = (V_1, \ldots, V_t)$, $P_2 = (W_1, \ldots, W_q)$, and $P = P_1 \cup P_2$.

We make the following observation on the Simpson scores of the alternatives before y is added.

- $Sim_P(c) = q - 1$. Indeed, $N_P(c, d) = q - 1$ and $\forall v \in \mathcal{V}$, $N_P(c, v) = q - 1 + d_I(v) \geq q$.
- $Sim_P(d) \leq 6 \leq q - 2$. This is because for any $v \in \mathcal{V}$, v is covered by the 3-sets for no more than $q - 2$ times (the assumption of the input X3C instance), which means that in P_1, $d \succ v$ for at most $q - 2$ times, i.e., $N_P(d, v) = d_I(v) \leq 6 \leq q - 2$.
- For any $v \in \mathcal{V}$, $Sim_P(v) \geq q$. Actually, $N_P(v, d) = N_P(v, c) = t - d_I(v) + q \geq 3q - 2 - (q - 2) \geq q$. Now, assume $v = v_i$. If $i < j$, then $N_P(v, v_j) = N_{P_1}(v, v_j) \geq t - d_I(v) \geq 2q - 2 - (q - 2) = q$ and if $j > i$, $N_P(v, v_j) = N_{P_2}(v, v_j) = q$.

Now, assume that $I = (\mathcal{S}, \mathcal{V})$ is a yes-instance of X3C; hence, there is a $J \subset \{1, \ldots, t\}$ with $|J| = q$ and $\bigcup_{j \in J} S_j = \mathcal{V}$. We show how to make c a co-winner by introducing one new alternative y.

- For any $j \in J$, we let $V'_j = [(\mathcal{V} \setminus S_j) \succ d \succ c \succ y \succ S_j]$.
- For any $j \in \{1, \ldots, t\} \setminus J$, we let $V'_j = [y \succ (\mathcal{V} \setminus S_j) \succ d \succ c \succ S_j]$.
- For any $j \leq q - 1$, we let $W'_j = [c \succ y \succ \text{rev}(\mathcal{V}) \succ d]$.

- Let $W'_q = [y \succ \text{rev}(\mathcal{V}) \succ d \succ c]$.
- Finally, let $P' = (V'_1, \ldots, V'_t, W'_1, \ldots, W'_q)$.

In P', the Simpson score of y is $q-1$ (via c), because $t = 2q-2$, which means that $t - q + 1 = q - 1$; the Simpson score of c is $q-1$ (via d); the Simpson score of d is no more than $q-1$ (via any of $v \in \mathcal{V}$); and the Simpson score of any $v \in \mathcal{V}$ is $q-1$ (via y). Therefore, c is a co-winner for the Simpson rule.

Next, we show how to convert a solution P' to the above PcWNA instance for the Simpson rule to a solution to the X3C instance. Let $P' = (V'_1, \ldots, V'_t, W'_1, \ldots, W'_q)$ with $P'_1 = (V'_1, \ldots, V'_t)$ and $P'_2 = (W'_1, \ldots, W'_q)$ be a profile such that c becomes a co-winner according to the Simpson rule when alternative y is introduced.

We make the following observations.

(a) $\forall v \in \mathcal{V}$, $N_{P'}(v, y) \le q - 1$,

(b) $N_{P'}(y, c) \le q - 1$ and $N_{P'}(y, d) \ge q$,

(c) $y \succ c$ in W'_q.

For item (a): Since c is a winner, we have that for any $v \in \mathcal{V}$, $Sim_{P'}(v) \le Sim_{P'}(c)$. Thus, using Property 2, $Sim_P(c) = q-1$ and $Sim_P(v) \ge q$. We have the following calculation.

$$\min\{N_{P'}(v, y), q\} = Sim_{P'}(v) \le Sim_{P'}(c) \le Sim_P(c) = q - 1$$

For item (b): First from (a), we deduce that for any $v \in \mathcal{V}$, $N_{P'}(y, v) \ge t + q - N_{P'}(v, y) > q$. Thus, we obtain:

$$Sim_{P'}(y) = \min\{N_{P'}(y, c), N_{P'}(y, d)\} \le Sim_{P'}(c) \le Sim_P(c) = q - 1 \quad (2)$$

Now, assume $N_{P'}(y, d) \le q - 1$. Then, $N_{P'_2}(d, y) = q - N_{P'_2}(y, d) \ge q - N_{P'}(y, d) \ge 1$. Hence, there exists $i \le q$ such that in W'_i, we have that for any $v \in \mathcal{V}$, $v \succ d \succ y$. Moreover, $N_{P'_1}(d, y) = t - N_{P'_1}(y, d) \ge 2q - 2 - (q-1) = q - 1$. Let $J_0 \subseteq \{1, \ldots, t\}$ (with $|J_0| = q - 1$) be the subscripts of arbitrary $q-1$ votes in P'_1, where $d \succ y$. Because $|\mathcal{V}| = 3q$ and $|S_j| = 3$, there exists $v^* \in \mathcal{V} \setminus \bigcup_{j \in J_0} S_j$. We deduce that for all $j \in J_0$, $v^* \succ y$ in V'_j. In conclusion, $N_{P'}(v^*, y) \ge |J_0| + 1 = q$, which contradicts item (a). Using inequality (2), item (b) follows.

For item (c): Otherwise, by the definition of W_q, we deduce:

$$\forall v \in \mathcal{V},\ N_{P'_2}(v, y) \ge 1 \quad (3)$$

On the other hand, using $N_{P'_1}(y, c) \le N_{P'}(y, c)$ and item (b), we have $N_{P'_1}(c, y) = t - N_{P'_1}(y, c) \ge t - N_{P'}(y, c) \ge t - (q - 1) = q - 1$. Let $J_0 \subseteq \{1, \ldots, t\}$ (with $|J_0| = q - 1$) be the subscripts of arbitrary $q-1$ votes in P'_1, where $c \succ y$. We have $\mathcal{V} \setminus \bigcup_{j \in J_0} S_j \neq \emptyset$ since $|\mathcal{V}| = 3q$ and $|S_i| = 3$. Hence, there exists $v^* \in \mathcal{V} \setminus \bigcup_{j \in J_0} S_j$ such that:

$$N_{P'_1}(v^*, y) \ge |J_0| = q - 1 \quad (4)$$

Summing up inequalities (3) (let $v = v^*$) and (4), we get obtain a contradiction with item (a).

From items (b) and (c), we get $N_{P'_1}(y, c) = N_{P'}(y, c) - N_{P'_2}(y, c) \le q - 1 - 1 = q - 2$. Thus, $N_{P'_1}(c, y) = t - N_{P'_1}(y, c) \ge t - (q - 2) = q$. Let J denote the subscripts of arbitrary q votes in P'_1 where $c \succ y$. We claim $\bigcup_{j \in J} S_j = \mathcal{V}$. Otherwise, there exists $v^* \in \mathcal{V} \setminus \bigcup_{j \in J} S_j$. It follows that for any $j \in J$, $v^* \in (\mathcal{V} \setminus \bigcup_{j \in J} S_j) \subseteq \mathcal{V} \setminus S_j$, which means that $v^* \succ c \succ y$ in V_j. Hence, $N_{P'}(v^*, y) \ge N_{P'_1}(v^*, y) \ge |J| = q$, which contradicts item (a). In conclusion, $I = (\mathcal{S}, \mathcal{V})$ is a yes-instance of X3C. Therefore, PcWNA is NP-complete for Simpson.

For the PWNA problem, we make the following change. Let $W_q = [\text{rev}(\mathcal{V}) \succ c \succ d]$. Then, before the new alternative is introduced, the Simpson score of c is q. Then, similarly we can prove the NP-hardness of the PWNA problem. □

7 Plurality with runoff

In this section, we focus on possible co-winners, which means that ties are never broken, neither in the first round nor in the second round. If a tie occurs in the first round, then all possible compatible second rounds are considered: for instance, if the plurality scores, ranked in decreasing order, are $x_1 \mapsto 8, x_2 \mapsto 6, x_3 \mapsto 6, x_4 \mapsto 5 \ldots$, then the set of co-winners contains the majority winner between x_1 and x_2 and the majority winner between x_1 and x_3.

Proposition 3 *Determining whether $c \in \mathcal{C}$ is a possible (co-)winner for plurality with runoff is in* P.

The proof does not present any particular difficulty, and due to the lack of space, we only give a very brief sketch for the PcWNA problem. It proceeds in two steps as follows. Let $\succeq_M^P$ be the weak majority relation induced by a profile P. Let P be a profile over $\mathcal{C}$. c is a possible co-winner in P if and only if one of the following two conditions hold:

1. There exists a completion P' of P such that c and some $d \in \mathcal{C} \setminus \{c\}$ are possible second round competitors, and $c \succeq_M^{P'} d$.

2. There exists a completion P' of P such that c and some $y \in Y$ are possible second round competitors, and $c \succeq_M^{P'} y$.

For each of these two conditions we can find equivalent, polynomial-time computable characterizations.

For the PWNA problem, the algorithm is similar: we need to make sure that the pairs of alternatives that enter the second round must be (c, d), where $c \succ_M^P d$.

8 Conclusion

In this paper we have gone much beyond existing results on the complexity of the possible (co-)winner problem with new alternatives. While [4, 5] focused on scoring rules, we have identified three new rules for which the PcWNA problem is NP-complete (Bucklin, Copeland, and Simpson). We also showed that the PcWNA problem has a polynomial time algorithm for plurality with runoff, and as far as approval voting is concerned, we have given three definitions of the extension of a profile to new alternatives and shown that depending on the chosen definition, the problem can be trivial or NP-complete. Our NP-completeness proofs and algorithms for the PcWNA problems can also be extended to the PWNA problems for approval, Bucklin, Simpson, and plurality with runoff. The results are summarized in the following table.

Voting rule	**PcWNA**	**PWNA**
Borda	P [5]	
2-approval	P [5]	
l-approval ($l \geq 3$)	NP-complete [2] [5]	
Approval	P (Definition 1) NP-complete (Definition 2) Trivial (Definition 3)	
Bucklin	NP-complete [2]	
Copeland_0	NP-complete [3]	?
Simpson	NP-complete [3]	
Plurality with runoff	P	

Table 1: Complexity of PcWNA and PWNA problems for some common voting rules.

[2]Even with 3 new alternatives.

[3]Even with 1 new alternative.

An obvious and interesting direction for future research is studying the computational complexity of the PcWNA (PWNA) problems for more common voting rules, including Copeland$_\alpha$ (for some $\alpha \neq 0$), ranked pairs, and voting trees. Even for Copeland$_0$, the complexity of the PWNA problem still remains open.

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